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Some More Thoughts on Placing the Decimal Point in Quotients*

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In the February issue of The Mathematics Teacher, Claude H. Brown has stated a very good argument for properly placing the decimal point in a quotient by the subtraction method.[1] Since this article presented arguments for the subtraction method, it might be well to state some arguments for the caret method as well as try to show that advocates of the caret method cannot necessarily be charged with mechanistic teaching.

Brown says,

> One obvious objection to the scheme of always transforming division problems in which the divisor is an integer is the mechanical nature of the process.[2]

Again,

> Probably the most significant factor in determining the choice of a method for locating the decimal point in division is the conception of the nature and educative function of arithmetic held by the individual making the choice. If arithmetic is to be taught merely for its utilitarian values, if the only educational purpose to be served by the teaching of arithmetic is that of developing in the individual the ability to compute with reasonable speed and accuracy, if an understanding of a given process is not essential, then use of a purely mechanical procedure like the caret method may be permissible.[3]

Some proponents of the caret method would take issue with the above statement "dubbing" the caret method as "a purely mechanical procedure." Those teachers who wish to make the educative process one of developing basic relationships and understandings can point out that it is merely a question of which one of two very basic mathematical principles are to be used as a basis for pupils placing the decimal point. Professor Brown wants to make the concept of inverse operations the basis. The concept of inverse operations in arithmetic should be developed but not necessarily as applied to placing the decimal point in a quotient. Proponents for the caret method base their case on another very important concept in mathematics, namely, the concept of ratio. More specifically, they can base their argument on the principle that any two terms in a ratio may be multiplied by the same number

* The subtraction method was formerly used in nearly all texts. Perhaps the most serious fault of this method was the very common error of not annexing zeros to the dividend, an incorrect answer resulting. Now practically all texts use some form of the caret method. It is highly desirable that one uniform method be used in all schools. Practically all experts in arithmetic favor the caret method.—Editor.

[1] Brown, Claude H., *Some Thoughts on Placing the Decimal Point in Quotients*, The Mathematics Teacher, XXXVIII: 78–80, February, 1945. Since this paper was written Dr. J. T. Johnson of the Chicago Teachers College has commented on Dr. Brown's proposals for placing the decimal point in quotients. See The Mathematical Teacher for May 1945.

[2] *Ibid*, page 79.

[3] *Ibid*, page 80.

without changing the value of the ratio. Of course the term "ratio" would not be used in a sixth grade, however, the teacher can develop this understanding using the terminology of fractions. Later on, the application of this principle can be transferred to the concept of ratio.

Briefly, here is a sound basis for the caret method. The teacher can recall for the class that while studying fractions such expressions as 6/3 were interpreted to mean $3\overline{)6}$. In fact, this will be done by any teacher interested in developing the interrelationships between two mathematical processes and, for the sake of an argument, the groups advocating the different methods, will have to consider only those teachers in the two "camps" interested in developing these relationships. The next step in developing an understanding of the caret method would be the intuitive development of the principle that the numerator and denominator of a fraction may be multiplied by the same number (not zero, of course, although this is not mentioned in a sixth grade class) without changing the value of the fraction. After having developed this principle while teaching fractions, the next step in teaching the caret method is obvious, namely, $3.5\overline{)17.5}$ is the same as $17.5/3.5=175/35$. Hence, to find the answer to $3.5\overline{)17.5}$ it is easier to work $35\overline{)175}$ or

$$3.5_\wedge\overline{)17.5}_\wedge$$

This, of course, is the teaching procedure stripped of all details so essential in any good teaching process.

Certainly the proponents of the subtraction method cannot argue that this principle and the interrelationships of the symbolisms advocated above are not important for a well-rounded understanding of the mathematical processes. If this is conceded, the argument then reduces to a question as to which of two important mathematical processes are to be emphasized in the teaching of the placement of the decimal point in the quotient. In considering the values of each method, it should be observed that the caret method approached as outlined above, gives the teacher an opportunity to (1) develop an understanding of the relatedness of decimal and common fraction notations, (2) develop a background which will be very useful when the topic of ratio is to be considered in later years, and (3) use a principle very essential to an adequate understanding of common fractions. The arguments for the subtraction method are given in the article under discussion.

It would seem then that granted a teacher is interested in developing the interrelationships of the mathematical processes, each method has a very sound educational basis. To teachers inclined to be mechanical both methods will be taught mechanistically and hence lose much of their educative value.

Brown also states,

> Furthermore, the caret method is accurate only if the figures in the quotient are placed directly over the proper digits in the dividend. This implies a degree of precision in writing figures which is difficult to obtain when the pupil is striving for speed for when his attention is centered on the conditions of his problem and not on the computation process.[4]

There are two replies to this argument. (1) There is common agreement among Morton,[5] Taylor,[6] Wheat,[7] and Wilson,[8] that placing the figures in the quotient directly over the proper digits in the dividend is advisable when beginning the study of long division. (2) Considering the frequency with which long division is used in out-of-school situations by a large per cent of the school population, it is ques-

[4] *Ibid*, page 79.

[5] Morton, R. L., *Teaching Arithmetic in The Elementary School*, Chicago, Silver Burdette & Co., 1938. Vol. I, p. 297.

[6] Taylor, E. H., *Arithmetic for Teacher-Training Classes*, New York, Henry Holt & Co., 1937. p. 92.

[7] Wheat, H. Q., *The Psychology and Teaching of Arithmetic*, New York, D. C. Heath & Co., 1937, p. 325.

[8] Wilson, Guy M. Stone, Mildred B., and Dalrymple, Charles L., *Teaching the New Arithmetic*, New York, The McGraw-Hill Book Co., 1939, p. 167.

tionable whether a high degree of speed should be one of our objectives in teaching long division. In fact, it may be questioned whether a high degree of speed in long division (and this applies to long division only) is so essential even for those who continue their study of mathematics in the high school and college. To get a correct answer within reasonable time limits should be sufficient for the most part.

To sum up the argument, it would seem that the difference is so far as the mathematical foundations are concerned reduces to a choice, on the part of the teacher, as to which mathematical principle is to be brought to the forefront while developing an adequate understanding of division exercises involving decimal fractions. Considering these facts, many teachers will probably continue to disagree with Brown's conclusion, which is

> Hence, as these facts indicate, the subtraction method of locating the decimal point in quotients is preferable and should therefore be taught either in addition to, or instead of, the caret method.[9]

[9] *Ibid*, p. 80.

A Method of Self-Instruction for Learning the Thirty-Six Addition Combinations with Sums from Eleven to Eighteen and Their Corresponding Subtraction Facts in Grade II

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In an earlier article,[1] the writer raised the question whether pupils in the primary grades are able to learn the facts of arithmetic largely by their own efforts. He reported a partial answer in a description of the successes of seventy-two pupils in Grade I with a method of self-instruction for learning the forty-five easier addition and subtraction combinations. These pupils learned the combinations through the study of groups. Three methods for studying groups were employed, namely, counting groups, comparing groups, and the analysis and synthesis of groups.

The present article describes the achievements of sixty-four pupils with a method of self-instruction for learning the thirty-six addition combinations with sums from eleven to eighteen and their corresponding subtraction facts in Grade II. These pupils were enrolled in ten one-teacher and four two-teacher schools for the school year, 1940–41 in Berkeley County, West Virginia. As mentioned above, they learned their arithmetic in Grade I by methods of self-instruction. In Grade II, they attempted to learn an appropriate course of arithmetic by similar methods. Herein certain preliminary preparations and the teaching procedures which were employed to assist the pupils in learning one part of the instructional program as prescribed for Grade II are detailed.

[1] D. Banks Wilburn, "*A Method of Self-Instruction for Learning the Easier Addition and Subtraction Combinations in Grade I*," Elementary School Journal, XLII (January, 1942), 371–380.

Preliminary Preparations

In order that the reader may view the content of the teaching program for Grade II as a series of interrelated learning activities, it seems appropriate to describe briefly the treatment of the content which was presented before the pupils attempted to learn the thirty-six addition and subtraction combinations. In Grade I, the pupils were led to form ideas of number by the study of groups. Now these ideas were extended and enlarged by providing exercises which assisted them to develop an understanding of the number ideas from ten to one hundred. A study of the number system was made in order to view it as a series of groups systematically planned with the standard group of ten as the basis.

A quantitative understanding of the many number groups was developed to the effect that the pupils described fifteen as one ten and five ones, twenty-five as two tens and five ones, and fifty as five tens. At this stage in their learning, they were introduced to methods which aided them to add and subtract tens. As a result of this study, the pupils were led to form the generalization that tens were added and subtracted just like ones. Next they concerned themselves with learning to add higher decade combinations. The facts used for these learning activities were selected from the one hundred sixty-one combinations which do not require bridging.

Later the pupils were directed to make a study of the higher decade subtraction

combinations because much practice was provided in using the easier subtraction facts in a different situation and on a higher level of understanding and use. Finally more practice was provided with the easier addition combinations through an introduction to the process of column addition. Here the pupils were led to give their attention to the various ways of rearranging a single group of ten or less into three separate groups.

The Teaching Procedures

With a background of number ideas which were developed through the study of groups to and including the group of ten, the pupils were ready to examine a new method of studying the arrangements of groups. This new method of studying groups which was employed to aid the pupils in learning the thirty-six addition combinations with sums from eleven to eighteen and their corresponding subtraction facts is described by Wheat[2] in a recent book. The teaching techniques proceed along five definite steps as follows:

1. Attention to arrangement, when the teacher makes the arrangement.
2. Attention to arrangement, when the pupils make the arrangement.
3. Attention to arrangement, when the pupils think the arrangement.
4. Attention to arrangement, when the objects are present only in imagination ("problem-solving," so-called).
5. Attention to arrangement, when no objects are present.[3]

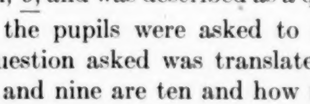
In the study of groups, the pupils dealt with combinations whose sums were greater than ten, such as, $\begin{array}{r}9\\ \underline{2}\end{array}$, $\begin{array}{r}8\\ \underline{4}\end{array}$, and $\begin{array}{r}8\\ \underline{9}\end{array}$. In dealing with such combinations they were not required to bring together, or think together, the two groups into a single group. They rearranged the two groups into a group of ten and so many more. Thus, to bring eight and nine together, the pupils combined the groups into a group of ten and a group of seven. Although the answer was said to be, "seventeen," it was understood to mean one ten and seven ones. When it was written, the pupils recognized that the numeral in ten's place represented one ten, and the numeral in one's place represented seven.

From the beginning, an effort was made to make certain that the pupils understood each necessary step in the rearrangement of two groups into a group of ten and so many more. A definite procedure was followed in order that the method of work would become increasingly useful in the study of each new arrangement. The teacher began the new study by directing the pupils' attention to two groups. Let us say, two groups of nine were presented to the pupils. The groups were represented thus:

The pupils were asked to count the dots or objects in each group. They were asked the question, "How many more than ten are nine and nine?" The addition was written, $\begin{array}{r}9\\ \underline{9}\end{array}$, and was described as a question which the pupils were asked to answer. The question asked was translated thus: "Nine and nine are ten and how many?"

The demonstration continued with the question, "How many are needed to make the group of nine a group of ten?" The pupils answered, "One."

The teacher cautioned the pupils to watch as she increased the nine to ten. A line was drawn to enclose the one group of nine, and one dot in the other group of nine. This rearrangement was illustrated, thus:



[2] Harry Grove Wheat, *The Psychology and Teaching of Arithmetic*, pp. 288–299. Boston: D. C. Heath & Co., 1937.

[3] Harry Grove Wheat, *Ibid.*, p. 294.

Now the pupils were asked, "How many dots are within the larger group?" "Ten."

"How many dots are left in the other group?" "Eight."

The pupils determined the answers by observing or counting the dots in each group.

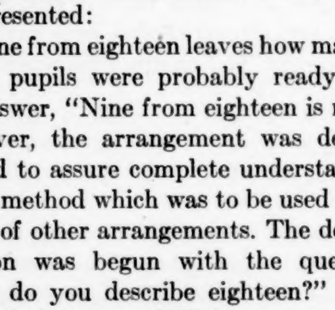
"How many are ten and eight?" "Eighteen."

"How many are nine and nine?" "Eighteen."

The answer, eighteen or ten and eight, was written, thus, $\begin{array}{r} 9 \\ \underline{9.} \\ 18 \end{array}$

The demonstration was repeated enough times to make certain that the pupils understood what had been done. Then the corresponding subtraction arrangement was presented:

"Nine from eighteen leaves how many?" Many pupils were probably ready with the answer, "Nine from eighteen is nine." However, the arrangement was demonstrated to assure complete understanding of the method which was to be used in the study of other arrangements. The demonstration was begun with the question, "How do you describe eighteen?" "One ten and eight ones." Eighteen was represented according to the pupils' description, thus:



Again the question was presented, "Nine from eighteen leaves how many?" While the answer was probably apparent, the discussion was continued as follows:

"If nine are to be taken from eighteen, from where are the nine to be taken?" The pupils were led to draw upon previous learning activities. They were able to compare a group of eight with a group of nine. It was known that nine was larger than eight. Therefore the pupils concluded that nine could not be taken from eight but that a group of nine could be removed from a group of ten. At this point, the teacher illustrated the withdrawal of the group of nine from the group of ten. In case the demonstration involved the use of dots on the blackboard, a ring was drawn around the nine dots as shown above.

Now the pupils were asked, "How many dots in the first group are outside the ring?" "One."

"How many dots are in the second group?" "Eight."

"One and eight are how many?" "Nine."

"Nine from eighteen is how many?" "Nine."

The answer was written thus: $\begin{array}{r} 18 \\ \underline{-9.} \\ 9 \end{array}$

The demonstrations which have been presented illustrate the first step in learning to deal with the thirty-six addition and subtraction combinations. In the second step, the pupils proceeded independently to work out the arrangements with the aid of objects according to the procedures of step one. As they gained facility in the use of the methods of work, they were encouraged to think the arrangements without manipulating the objects. To assist the pupils in this, the third step, many arrangements were described orally. Later they were introduced to work at the level of step four, with the use of objects in the imagination only. Here verbal and written statements were used to describe the arrangements. In step five, the arrangements were presented as combinations with the answers to be written without the assistance of specific aids. The act of making the arrangements was intended to be one of thought entirely.

The Effectiveness of the Teaching Procedures

The effectiveness of the teaching procedures was measured through the administration of a test, and through a series of interviews with pupils. The test was con-

structed so as to include all the processes which were considered in the instructional program for Grade II. It contained ninety-five addition, seventy-six subtraction, twenty-three multiplication, and twenty-three division facts. Another section presented twenty-five one-step problems.

The addition combinations were presented in the following manner. Ten addition facts whose sums ranged from six to ten were selected at random. They were repeated in twenty combinations which involved the addition of tens. In these items, the simple arrangements with sums from six to ten were used in both ten's and one's place. The thirty-six addition combinations with sums from eleven to eighteen were present in another section, and from these combinations, ten facts were selected at random for presentation on a higher level of use. They were as follows:

7	8	9	8	9	5	6	8	9	9
4	5	5	4	7	7	9	8	8	9

The combinations were presented with the ten addition facts whose sums ranged from six to ten. By placing the ten facts, which are listed above, in ten's place, the pupils were required to write answers to these items:

54	71	85	94	64	82	82	96	97	94
72	45	53	53	94	85	47	73	93	86

The items in the subtraction section of the test were the opposites of the addition facts. The exercise in multiplication presented the twenty-three multiplication combinations with products from four to eighteen. The twenty-three division combinations with dividends from four to eighteen were selected as the content of the exercise in division. The twenty-five problem statements presented combinations which occurred previously in the other four sections of the test. Upon request the writer will make available a copy of the test described above, and the directions for administering same to the reader.

During the latter part of May, 1941, the test was administered to sixty-four pupils in Grade II. When distributions of the pupils' scores were made for the sections of the test which presented the thirty-six addition combinations with sums from eleven to eighteen and their corresponding subtraction facts, it was revealed that the combinations had been learned well enough to produce mean scores of 35.3 and 33.8 for the addition and subtraction facts respectively. The mean scores indicate that the pupils with few exceptions made perfect or nearly perfect scores. Table I shows the number of pupils who attained such success.

TABLE I. NUMBER OF PUPILS MAKING PERFECT AND NEARLY PERFECT SCORES IN THE THIRTY-SIX ADDITION COMBINATIONS WITH SUMS FROM ELEVEN TO EIGHTEEN AND THEIR CORRESPONDING SUBTRACTION COMBINATIONS

Type of Test	Number Scoring 36	Number Scoring 30–36
Addition	51	63
Subtraction	45	60

There are many outcomes of the learning process which cannot be described by the analysis of test results and other objective data. Evidence of the existence of these outcomes must be gathered by observing signs of the development of attitudes, habits, and understandings. Since the methods of learning the number facts emphasized the understanding of certain ideas which were involved in the processes, one became interested in having some indication of the formation of number ideas and relationships. For the purpose of collecting such data, the writer planned and conducted individual interviews with a number of pupils. When a pupil was interviewed, the object of the discussion was to ascertain the presence of specific ideas which were associated with a particular arithmetical process. Two interviews are recorded herein. They seem to indicate that the teaching procedures did not neglect the development of the ability to

understand what was involved in a process.

Interviewer: "Can you tell me how to find the answer, if you didn't know, to the addition, $\begin{array}{r}9\\ \underline{5}\end{array}$?"

Pupil: "I can think nine and one make ten. Ten and four make fourteen."

Interviewer: "From where do you get the one?"

Pupil: "I get one from five, and four are left."

Interviewer: "What do you see in this addition, $\begin{array}{r}71\\ \underline{45}\end{array}$?"

Pupil: "I see one and five, and seven tens and four tens."

Interviewer: "Do seven tens and four tens look the same as anything you have seen before?"

Pupil: "They are just the same as four tens and seven tens, and they look just like seven ones and four ones."

Interviewer: "How do you add seven tens and four tens in the addition, $\begin{array}{r}71\\ \underline{45}\end{array}$?"

Pupil: "Just as I add seven and four."

The remainder of the interview was concerned with a general discussion relative to the pupil's ability to recognize number situations when presented in written statements.

Interviewer: "Why were you able to work all of the problems in the tests?"

Pupil: "They were easy."

Interviewer: "Did you have any trouble in reading the problems?"

Pupil: "No."

Interviewer: "What made the problems so easy?"

Pupil: "I have done the same kind of problems before."

Interviewer: "What do you think about when you read the words, (How many, and Together)?"

Pupil: "They tell me to add."

Interviewer: "Do you like to study arithmetic?"

Pupil: "Yes."

Interviewer: "Why?"

Pupil: "Because I like to work."

The next interview illustrates a pupil's understanding of the process of subtraction. The conference proceeded as follows:

Interviewer: "How do you think the answer to the subtraction, $\begin{array}{r}14\\ \underline{-8}\end{array}$?"

Pupil: "You take eight from ten. Two are left. Put the four with the two, and you have six left."

Interviewer: "Why don't you take eight from fourteen?"

Pupil: "You can't take eight from four."

Interviewer: "Yes, but you can take eight from fourteen."

Pupil: "You can't take eight from fourteen that way until you think eight from ten."

Interviewer: "What is fourteen?"

Pupil: "Fourteen is one group of ten and four ones. That is why you can't take eight from four. Eight is larger than four."

Interviewer: "How do you think this subtraction, $\begin{array}{r}158\\ \underline{-62}\end{array}$?"

Pupil: "You take two ones from eight ones. Six ones are left. Then, you think six from fifteen."

Interviewer: "What are the differences in doing these subtractions, $\begin{array}{r}15\\ \underline{-6}\end{array}$, $\begin{array}{r}8\\ \underline{-2}\end{array}$, and $\begin{array}{r}158\\ \underline{-62}\end{array}$?"

Pupil: "The only difference is in $\begin{array}{r}158\\ \underline{-62}\end{array}$, ones and tens are subtracted from ones and tens. It is done just like two examples which tell you to take away ones."

Interviewer: "Do you like to study arithmetic?"

Pupil: "Yes. It is easy."

Interviewer: "Why is it so easy?"
Pupil: "I know how to think out what I do."

SUMMARY

This article has described the methods of learning which were used to learn the the thirty-six addition combinations with sums from eleven to eighteen and their corresponding subtraction facts by sixty-four pupils in Grade II. Data have been presented to indicate that the pupils learned successfully both sets of combinations, and they found one set no more difficult to learn than the other. Moreover, the evidence seems to reveal that the methods of learning encouraged the pupils to develop an understanding of the number system.

Possible Articulation for the Junior High School with the Elementary School and the Senior High School

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The job which faces mathematics teachers in the years just ahead promises to become an ever increasingly important but difficult one. In a world where quantitative situations must be dealt with again and again accurately and with great precision, mathematical literacy is equally as important as the ability to read and write—to express oneself with clarity and conviction.

The schools must insure this mathematical literacy to all who can possibly achieve it. The need for mastery of skills in simple computation, and for control and intelligent use of certain basic mathematical concepts and principles is universal. It is our obligation to furnish this training to all who will avail themselves of it.

Furthermore, we are entering upon an age of scientific and technological culture unparalleled in the World's history. Many of the young people in our class rooms today should be—and will be—leaders in this new era of living. For these young people we must provide training for mathematical mastery and power which will enable them to assume this leadership.

The task before us is a stupendous one. It is, however, a challenge which mathematics teachers must accept. And it cannot be the work of leaders in the field of mathematical education alone. Its maximum achievement can only be the result of all teachers of mathematics working together—teachers from all levels of mathematical education.

The road to mastery—or even competency—in mathematics is a long one, and progress thereon should be continuous. Yes, growth toward the mastery of mathematics should be continuous. It should have its foundations well-laid in the early grades of the elementary school. It should progress naturally and smoothly—level by level—in unbroken sequence throughout all subsequent educational experiences of the individual.

This progress can best be realized only where some sort of educational co-ordination, articulation or integration is maximally employed. Many school systems and other educational groups are achieving much in this direction. Their excellent educational attainments bespeak the worthwhileness of such procedure.

Let us define these procedures and then choose the one which seems most nearly ideal. To co-ordinate is to combine into harmonious relation or action—to bring about harmonious adjustment between two or more distinct entities. Integration is a process of summation. It implies a combination of elements so intimately connected that a perfect whole results or is approached as a limit. Usually it suggests a complete fusion or coalescence of particulars with a loss, therefore, of their separate identities. Articulation also implies as its result a perfect whole, but it differs from integration in that it implies no loss of identity or of distinctness of the things combined. Each part fits into another in a manner comparable to the fitting into each other of two bones at a joint. A structure is built up that functions as a whole, yet without loss of flexibility or distinctness in any of its component units or without any conflict between them.

Is not articulation our best procedure in bringing about the continuous growth so necessary for satisfaction and success in the sum total of mathematical education. Assuming this to be true, let us determine

the place of the junior high school in the educational structure we are building.

The junior high school is in an intermedial position between two very distinct educational entities—the elementary school and the senior high school. The procedures and techniques on these two levels of education are of necessity quite different. The age of the child, as well as the subject matter taught, make this difference necessary. The junior high school must tie the two together into an hormonious whole. To bring about this articulation a four-fold responsibility devolves upon the junior high school.

1. It must provide an adequate and natural continuance of the work of the elementary school.

2. If at all possible, it must correct all mathematics retardation and shortages existing any of its pupil personnel. We are told such adjustments must be made here; it is virtually impossible to make them later on.

3. The junior high school must provide an expanding and deepening experience with the problems of everyday living.

4. It must strengthen and extend the foundations for subsequent experiences with mathematics.

The junior high school program of mathematics must be planned as a whole, built around certain broad categories. The organization and arrangement of the units, which go to make up this whole, may vary in order to provide for local and individual needs and interests. But the following categories must be universally accepted by all junior high schools:

1. Number and computation—a mastery of skills, an expanding appreciation of quantitative values and inter-relationships.

2. Measurement and informal geometry—a continuance of the rich experiences in direct measurements of the Elementary school—and a discovery of concepts and principles to be used later on in the more formal geometry of the senior high school.

3. Construction and interpretation of graphs—including varied and rich experiences in statistical graphing, map reading—and an introduction to the study of function graphs.

4. Introduction to the functional core of algebra through formulas and equations.

5. A generous application of the various phases of mathematics to the problems of everyday living.

The junior high school teacher should be broadly educated in her own field and in related fields of learning. She should be very versatile in her reactions to situations which arise. She should not only know her own level of mathematical education, but she should also know the levels directly below and above her own.

Meaningful teaching represents a building-up process in which old ideas work together in new situations. What often appears to be complicated and difficult for pupils, could in reality be very simple if they possessed the proper educational foundation and if that foundation were appreciated and properly built upon in subsequent learning experiences. Maturation of ideas, skills and concepts will result only from an instructional program so organized as to contribute to the continued development of understanding and insight into mathematical relationships as the pupil advances, stage by stage, throughout his educational experiences.

The elementary school is an activity school. Life situations, on the level of the child, and within his range of interests, are set up in configurations so planned as to teach the lessons desired. His interest having been aroused, the child is led to discover the number challenges and the need for mathematical manipulation.

He sets down his discoveries and determines a simple pattern of reasoning to be followed. He then applies this pattern and works out his little problems.

This, to a large extent, must be the procedure in the early months of the child's junior high school experience. He is not yet ready for abstractions and generalities. These must come gradually, as

his understanding deepens and his experiences expand.

The point at which the unique teaching in the junior high school begins, is the point at which the students' former knowledge ends. At the outset, the seventh grade teacher acquaints herself with her classes as she receives them from the elementary school. Having studied their general records, she proceeds to investigate individual cases, to diagnose individual and group difficulties from past testing records and personal history data submitted to the junior high school by the elementary school. She discovers many of these children have not attained the full measure of their ability in mathematical experiences. Thus deficiencies of previous school work must be made up. Remedial work must occupy an important place in the program of procedures. But this remedial work should not be super-imposed. It must not be meaningless drill. The child should be led to discover his own shortages and to want to do something to correct them.

What better unit with which to make this discovery than a unit of direct measurement! Children love to measure things. They have had much experience with such activity throughout the elementary school. They began measuring in the third grade, using rulers laid off in inches. Each subsequent year has added one step to their precision of measurement. The seventh grade finds them ready to measure by sixteenths of inches and to use the decimal ruler.

Measurement in the seventh grade should begin with the more familiar objects in the child's environment. Careful estimation should precede each measurement. Records should be kept of estimated and measured values. These should, in turn, be compared with an accepted measured value to determine accuracy to judgment and precision of measurement. Problems involving measurement should be brought to the mathematics classes from the Manual Arts classes, hobby shops, or from any other measurement activities within the experience of the child.

All measurements should be made to the degree of accuracy set up as a standard for this grade. Aids to precise measurement must be taught and considerable practice afforded in their appliction.

The applications of individual measurements often requires rather difficult computation with common and decimal fractions. Some children find themselves not sufficiently proficient in these computational skills to permit the successful manipulation of the measurements they have made that are required in the practical applications. They are not surprised to discover that they have been hampered by their computational inefficiency. They have associated number skills with measurement units throughout most of their school experiences.

These children are now ready to make up the shortages which they have found to exist. They feel a need for directed practices in the skills which they have not mastered. So the teacher re-teaches, emphasizing and clarifying concepts. The children hear these principles stated and re-stated, and they are guided in expressing them and in applying them as appropriate occasions arise. Thus understanding makes practice meaningful; generalizations become useful. The children begin to acquire a science of number upon which to build their subsequent structure of mathematical education.

Children for whom no shortages have existed are exploring new fields, extending their knowledge and appreciation of quantitative values and relationships.

Shapes in nature, objects and designs in man-made environment, are studied and compared. The child observes both size and shape as determining characteristics of things about him. He learns to associate linear and angular measurement with size, shape, and a new concept—an appreciation of angles and their significance becomes functional to the learner. There follows a period of experimentation with lines and angles through simple geo-

metric constructions. Geometric principles and concepts are discovered, clarified and applications made.

To provide practice in these applications, and to develop greater skill in the use of such mathematical instruments as the compass, straight edge and protractor, the children are guided in the study and construction of simple geometric designs, and scale drawings.

As an outgrowth of all this number study and measurement experience, the child not only grows toward a mastery of skills; his appreciation of number relationships also deepens and expands. He has long understood the concepts "how much more?" "how many times?" "what part of?" His experience with this type of thinking began in the third grade and has been growing continuously since. It is not difficult then, for the seventh grader to understand the simpler implications of ratio and to use them in problem situations involving comparison of sizes and values. Scores of such interesting problems are available within the range of the child's experience. They can be built into very interesting configurations.

These boys and girls are becoming increasingly aware of the social significance of mathematics—its utility and application in the affairs of daily living. The wise teacher capitalizes upon this awareness, and takes steps to develop in these children the ability and disposition to view the affairs of life in an orderly, systematic manner. She teaches them to see relationships involved in daily problem situations and to use quantitative techniques in their solution.

Problem solving is not new to the seventh grader. He began solving very simple word problems in the third grade. Each subsequent year has added just a little to his experience with reasoning situations. This background should be recognized and built upon.

Procedures in the seventh grade, however, must be increasingly formal and thorough, affording strong foundation for the abstractions and generalities in problem solving which will confront the students in the mathematical experiences just ahead.

First of all, the student must learn to read his problem carefully to determine exactly what is given and what is required.

He should then list the data given and the question to be answered.

He should visualize the problem situation, whever possible, by drawing an appropriate sketch.

He should select relevant data and discard the irrelevant.

He should analyze the relationship of quantitative values and determine procedures to be followed.

He should always judge a reasonable answer in advance of computation.

He should compute carefully and check all results for common sense and accuracy.

He should test his result in terms of the question asked at the outset.

This procedure, followed faithfully throughout the seventh and eighth grades, should bring results that will greatly reduce difficulties in the ninth grade algebra and the mathematics of the senior high school.

Of equal importance in the mathematics program of the junior high school is the work with per cents. We must concede this work to be entirely new to the seventh grade child. It presents a new language and a comparatively new mode of expressing quantitative relationships. But the language is readily learned, if associated with the child's immediate environment. The relationships become basically simple when interpreted through the already familiar concept of ratio. This close association between ratio and per cent facilitates the acquirement of the new concept —per cent and deepens and expands the general concept of ratio. This is most important. For ratio is basic and vital to many mathematical experiences which are to follow.

Techniques in the use of per cents must

be developed. Their early mastery is directly commensurate with skills and understandings already acquired in the use of common and decimal fractions.

Concomitant with the mastery of techniques in percentage, is an appreciation of its many uses, and a growing ability to apply these uses in the management of daily affairs. Thus, percentage becomes an important medium of understanding in situations of social significance.

As the child advances chronologically and educationally, his horizon expands; his interests deepen and increase. The junior high school boy and girl of grades eight and nine experience a period of intensive interests. The affairs of today's environment are strongly significant to them and ideas concerning their future participation in the world of adults attract and demand their attention.

Many of these young people are already looking ahead to College, or the technical job which promises to make such an important contribution to their future happiness and success. They are beginning to realize how very vital to the realization of their ideals, is a generous mathematical education. So we plan for them a more formal—an enriched program of mathematics.

While this program must remain within range of the current interests and activities of the student, it will also reach out to the simpler generalities and abstractions heretofore not experienced by him.

Once again measurement must occupy an important place in the mathematics program of study. We are living in an age of measurement—of high precision. This is a fact we cannot impress too strongly upon the minds of Young America.

At the beginning of the year, the eighth grade student resumes practice in estimation and measurement. He begins to appreciate the science of measurement; that no measurement can be exact; the degree of precision to be attained depends upon the specific situation at hand and upon the instrument to be used. He learns the meaning of tolerance and comes to appreciate its significance in measurements of the machine shop and industry. He becomes acquainted with precision instruments heretofore strange to him.

Again many students find their progress hampered by shortages in computational skills. They come to realize the importance of meaningful practice in whole numbers, common fractions, decimals and per cents. No skill can be maintained—much less strengthened—without continuous practice. And mastery of these computational skills is so vital to success in this technological age.

In this re-emphasis on Arithmetical skills, stress should be laid on the generalities involved—the abstract principles fundamental to all computational processes. An understanding of these principles—an ability to express and apply them will be invaluable, later on in similar situations involving the use of Algebraic symbolism and notation.

The Junior High School student has learned much about quantitative relationships and inter-dependence of values. These concepts have been growing and expanding in his consciousness since the early days of the elementary school. In the seventh grade, he has learned to picture certain numerical relationships—simple statistical data—by using bar, circle and line graphs as the occasion prompted. The eighth grade student reviews these principles and practices of graphing. He collects and organizes interesting social data and pictures by the use of appropriate graphs. He learns to interpret changing values by means of the statistical graph. All of this involves much manipulation with common fractions, decimals, and per cents. It affords enriched experience with ratios and approximate values drawn to scale.

If changing values are continuous and uniform, and dependent one upon the other, the student is led to discover there must be an underlying principle controlling this change. He learns to write this

principle as a formula and to portray its significance by table and simple function graph.

This leads to a re-emphasis upon problem situations involving formulas with which the student is already familiar—formulas governing the perimeters and areas of the more common plane surfaces. Eighth grade students have had many successful experiences with these formulas in the seventh grade. They take quite readily to the newer principles which they are now led to discover—very simple examples of related change.

This new thinking is brought about through guided experimentation and observation. Most of it concerns the rectangle and the square. The student starts with a square of convenient size. Keeping one dimension the same, he varies the other by some easy ratio. He observes the effect of this variation upon the perimeter. He then varies both dimensions by the same ratio, and observes the effect upon the perimeter. He repeats these experiments several times using a different ratio of change each time. He then performs these experiments all over again, this time noting the effect upon the areas. He notes that area and perimeter are not proportionately affected by these variations. The more alert student looks for a reason for this discrepancy and finds it in the structure of the formulas involved. He is led to see the area as a product and the perimeter as a sum. Naturally multiplied change is more rapid than added change.

Intrigued with these discoveries, he is eager to carry the experiment still further. Again we start with a square—a 6-inch square. This time we keep the areas constant and vary both dimensions, noting the resulting inverse variation of these dimensions. We observe the effect upon the perimeter. We continue our experiment through many steps in variation. This poster shows the results of part of our experiment. We couldn't find cardboard large enough to record all our observations. We finished our measurements on the very longest stretch of black boards in the class room. Our last rectangle was 288 inches long and $\frac{1}{8}$ inch wide. How rapidly the perimeter increased in value, when the shorter linear dimension dropped below one inch in value! And now for a summary of observations. All areas were checked to be the original 36 sq. in. The perimeter had increased from 24 in. to $576\frac{1}{4}$ inches. The rectangle was rapidly approaching the appearance of one straight line. It was just a little difficult for these children to comprehend that this rectangle never would become one line no matter how long we continued our study of variation—that the 36 sq. in. of area would still exist.

The next experiment also was begun with a 6-inch square. This time we held the perimeter constant and varied the linear dimensions. Again the rectangles changed shape becoming long and narrow and approaching a straight line as a limit. The area became smaller and smaller.

One other observation was made from these experiments. Among all these rectangles, the square was found to have the greatest possible area for the least length of perimeter.

Also of very great importance in the eighth grade course of study is the unit on triangles. Time will permit only a cursory reference to this study. It deserves far more, for it makes such a rich contribution to the formal mathematics of later years.

This triangle study should include,

1. A clear concept of the triangle with its six component parts.

2. An appreciation of angle distribution and total angle value.

3. Classification of triangles as to line and angles.

4. An understanding of the properties of congruent and similar triangles, and skill in their use as tools for indirect measurement.

5. Indirect measurement through triangles drawn to scale.

6. The wind triangle in very simple aviation problems.

7. The right triangle, including
a) The rule of Pythagoras
b) The special 3–4–5 rt △, the isosceles rt △ and the 30–60–90 rt △.
c) The tangent function.

This unit cannot be satisfactorily completed without a generous experience in field measurements to illustrate and vitalize the meanings acquired.

A very appropriate study with which to close this paper is the study of solids. It can at once be utilitarian and technical. It builds upon many familiar concepts acquired as early as the elementary grades. It points the way to the abstractions and generalities of solid geometry in the senior high school.

First of all, the student learns to construct models of the solids to be studied. As he does so, he notes characteristics which cause these solids to assume their respective shapes. He then learns to group and classify these solids into three great families—the prisms, the pyramids, and the polyhedrons. As he does so, he notes similarities which make this grouping possible—the rectangular lateral surfaces of all prisms; their two bases in the shape of parallel and congruent polygons—the congruent, isosceles triangles which make up the lateral surface of all pyramids; the paramid's one base—the many congruent regular polygons which form the surfaces of polyhedrons.

The recognition of these relationships is most important. It makes subsequent learning experiences so much more meaningful.

The student has long been familiar with the rectangular prism. We build upon and extend that acquaintance until the student has acquired a genuine appreciation of its properties; until he shows skill in applying these properties to many, many problems of daily living.

Having learned the properties of one of these related solids, the work with all others becomes simple.

The junior high school is becoming increasingly aware of the responsibilities which devolve upon it in the over-all program of mathematical education. The schools are obligated, as never before, to insure mathematical literacy to all who will avail themselves of it. They must provide training toward mastery and power for all who desire it and will work for it.

But the road to mastery in mathematics is a long one and progress thereon must be continuous. It should have its foundations well-laid in the early grades of the elementary school. It should progress naturally and smoothly—level by level—in unbroken sequence, throughout all subsequent educational experiences of the individual in order to provide a continuous program.

This continuity can best be realized only where educational articulation or coordination is maximally employed. Because of its intermedial position between the elementary school and the senior high school, the junior high school must assume much responsibility for the success of such a program. Leaders in junior high school mathematical education are recognizing this challenge and are studying to meet it. Much research and study is now being undertaken and plans for procedure being set up.

When this program of articulation is being optimally used, I believe we shall have progressed far in meeting the challenge now before us.

Why Not Be Sensible About Meaning and Applications?

By Walter Bernard
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Several articles have been published recently in The Mathematics Teacher on the teaching of meaning. Two of these articles* appear to take opposite sides. Viewing the subject of meaning as a classroom teacher and a former engineer, I am of the opinion that the writers of these articles are both presenting theses of value to us who teach in the elementary and secondary schools. Now I am not stepping into the controversy between Buell and Wheat in the role of peacemaker. Perhaps by the time I am through this paper, both Buell and Wheat will be turning their guns, or pens, on me. I really believe that it is of the utmost importance that we should consider what part meaningful teaching should play in elementary and secondary mathematics.

The teaching of meaningful mathematics is the principal reason for the existence of mathematics as a separate department in secondary education. If our chief purpose is the teaching of mathematical applications, then we should let the commercial, shop, and science teachers teach what mathematics the pupils need. Mathematics as a separate department above the sixth grade could not be justified, except for pupils who wish to present algebra and geometry for college entrance. A separate mathematics department is based on the premise that there are facts and principles about numbers and magnitudes that warrant separate study. This knowledge of pure mathematics enables the pupil to apply mathematics efficiently to the solution of practical problems.

* Buell, Irwin A., "Let Us Be Sensible About It," The Mathematics Teacher, Nov., 1944, pp. 306–308.

Wheat, Harry G., "Why Not Be Sensible About Meaning?" The Mathematics Teacher, Mar., 1945, pp. 99–102.

In planning what is to be taught in mathematics, the educator should consult the work of not one group of specialists but of several. The experts should include (1) commercial, industrial, and scientific users of mathematics; (2) mathematicians; (3) psychologists; and (4) educationists. The direction which our courses in mathematics would take would be determined largely by what is learned from the users of mathematics; the structure and potentialities of our courses, from the mathematicians; the functioning of the child mind, from the psychologists; and efficient teaching procedures, from the educationists.

In the distant past the mathematicians were the only people consulted in the planning of a mathematics course. If one examines elementary and secondary school mathematics courses today, he can still find evidence that mathematicians had something to do with their content. Later some attention was paid to the users of mathematics; and we can see where navigation, commerce, farming, and the mechanic arts fade in and in some cases slowly fade out of the picture. Since 1910 the effect of the psychologists and educationists has been felt. Courses have been modified or even dropped out of the curriculum of some pupils. Topics have been shifted up a grade and even down a grade. Teaching methods, especially in the elementary school, have been changed. The educationists have asked us to make our courses more practical, directing us back to the users of mathematics. Lately the war has influenced our secondary school mathematics by causing the inclusion of topics on ballistics, communication, and aviation, and in some cases whole courses in these subjects.

One good result of attention to the applications of mathematics has been the observance that one is limited in his application of mathematics if he is not fully appreciative of the meaning of numbers, their properties, and the principles of their combination.

If we desire help in the teaching of meaning, the first place for us to go is to the works of the pure mathematician. According to him, mathematics is a logical framework built up from a foundation of undefined terms, definitions, and postulates upon which we merely hang or support those applications to which these postulates apply. Mathematics when well understood contains within itself the reasons for the steps taken in the solution of a practical problem. A good workable definition of mathematics is that it is organized and tested common sense.

To illustrate what has been said about mathematics, let us imagine Mr. Mathematician orally testing Mr. Mechanic on his knowledge of simple arithmetic.

"If I were to have you cut this 6-inch rod," asks Mr. Mathematician, "into 2-inch pieces, neglecting the width of the hack saw blade to make the problem simple, how many pieces would you get?"

"Three," promptly replies Mr. Mechanic.

"Why?" asks Mr. Mathematician.

"Because 6 divided by 2 is 3," retorts Mr. Mechanic somewhat impatiently.

Mr. Mathematician beams. He even laughs when he hears Mr. Mechanic say under his breath, "Fathead! Any fool knows that!"

Mr. Mathematician is greatly pleased to hear that Mr. Mechanic, in common with most people, has a logical grasp of whole numbers. Mr. Mechanic would probably characterize a foolish person as one who does not know that 2 and 2 make 4.

Mr. Mathematician decides to test Mr. Mechanic on fractions.

"Suppose that instead of cutting the 6-inch rod into 2-inch pieces," he asks, "you cut it into $\frac{1}{2}$-inch pieces. How many pieces would you have then?"

Mr. Mechanic does not reply so promptly this time. He stares off into space, his lips are tightly pressed one moment, and his mouth is open the next. At times his throat moves. After a pause of some length, he gives the solution.

"Twelve pieces."

"Good," says Mr. Mathematician. "Why?"

"Well, you see if I were to cut the 6-inch rod up into 1-inch pieces, I would get 6 pieces, and if I cut each 1-inch piece into $\frac{1}{2}$-inch pieces, I would get two $\frac{1}{2}$-inch pieces for each 1-inch piece; and since I had six 1-inch pieces, there would now be 12 pieces."

Mr. Mathematician shakes his head sadly.

"Good common sense," he remarks to himself, "but clumsy and not neat direct mathematics. I hate to think of his mental agony if I were to ask him to consider cutting the rod into $\frac{3}{4}$-inch pieces."

Inquiry discloses that Mr. Mechanic had a good grammar school drilling in fractions. He, in common with many others, knows how to divide by a fraction, but the drilling did not reveal to him the meaning of the process, and, therefore, he does not recognize an opportunity to apply the process to the solution of a simple practical problem.

It would have been a great day for Mr. Mathematician, if Mr. Mechanic had replied, "Twelve, because 6 divided by $\frac{1}{2}$ is 12. That's why."

If anyone thinks this story is farfetched, let him ask an adult friend how much is 6 divided by $\frac{1}{2}$, and should he succeed in drawing out the answer 12, ask his friend to justify the quotient. Why is not the quotient 13 or some other number? He will find many intelligent people who do not understand division and the application of fractions. For that matter there are textbooks on arithmetic that suggest clumsy methods of solving problems in order to avoid the use of fractions.

In advanced courses in mathematics,

mathematicians tell us that the fundamental processes originate in counting. Since adding is the simplest process, almost as simple as counting itself, the other three processes, subtraction, multiplication, and division, are explained in terms of addition. The quotient, in one of its meanings, tells us how many terms each equal to the divisor must be added up to yield a sum equal to the dividend. In the problems proposed to Mr. Mechanic, he was first asked in substance how many twos add up to six, and then how many halves add up to six. Both problems when so analyzed suggest division as the method of solution.

Does this mean that elementary and secondary school mathematics should be taught on a completely logical basis, with the courses built up from undefined terms, definitions, and postulates? That is exactly what teaching for complete understanding would mean. I believe that mathematicians would be the first to disapprove such an attempt. In several colleges it was found that freshmen could not study algebra on that basis. A college professor told me that he had attempted to teach Euclid's geometry on a 100% logical basis, but had had to give it up because it took him six weeks to reach the theorem that all right angles are equal. Furthermore, mathematicians, I think, would feel that much of the value of their work is lost, if those who are to apply mathematics hesitate to use its time-saving theorems.

Even if we do not organize our courses on a rigorously logical basis, there is still much room for meaningful teaching. That is the moral of the story of Mr. Mathematician and Mr. Mechanic. Let us look at the psychology of problem solving and computation as viewed by the mathematician. He has reduced it to the recognition of opportunities for applying mathematical theorems; for example, that an equation can be formed by expressing one quantity in two mathematically independent ways; that multiplying both terms of a fraction by a number does not change the value of the fraction; and, of course, many others. When a problem-solver cannot see how to apply a mathematical theorem, he resorts, for the time being, to common sense.

The last statement may sound as if I disapprove of common sense. Of course, I do not. I am using the term, "common sense," to mean reasoning from fundamentals. It is a slow process, and it is very easy for one to leave out an important step in his reasoning and arrive at a wrong conclusion. To protect ourselves against fallacy is the principal reason for proof in mathematics. A captivating aspect of mathematics is that the truth of some of its theorems is not readily apparent until subject to proof. When we use the theorems of mathematics we are enjoying the fruits of organized and tested common sense.

It might be well to point out here that in the solution of problems we also make use of theorems that originate not in mathematics but in the field of application. It is in accordance with a rule of commerce and not of mathematics alone that one concludes that if 1 article costs 5 cents, 6 articles should cost 30 cents. In fact, you may get the 6 articles for 25 cents. If a man agrees to work for $1 an hour and he works 10 hours in one day, he must be paid not $10, but $11; for according to the National Wage and Hour Law he is entitled to a time and-one-half rate for all time in excess of 8 hours.

From the mathematicians we learn the value in problem-solving of a knowledge of the structure of pure mathematics, but they also warn us of difficulties in imparting this knowledge by rigorous logic. When we turn to the psychologists and educationists, we are told more about these difficulties. In fact, the classroom teacher is aware of many of them from his own experience with children. Most children enjoy doing simple tasks and are satisfied with the teacher's approval as their only reward. Bright children are more interested than dull children in the applications

of what they learn in mathematics. While calling pupils' attention to applications made by engineers of various theorems of geometry, I have noticed the rapt attention of the brighter students and the bored attitude of the others. Most pupils below the age of twenty are not so much interested in "why" as in "how." Of course, there are exceptional children; but the great majority of "why's" directed at me turn out to be either attempts to stall the lesson or attempts to find out "how." Most of us are familiar with children who, when asked by their mother to go to the store, counter with "Why?" When I have asked pupils for the reason why 6 divided by $\frac{1}{2}$ is 12, they usually reply that is so because you have to invert the $\frac{1}{2}$ and then multiply by 6. To them "how" is "why." I am not so sure but that the children are sensible in being more interested in "how" than in "why." Of the two, "how" is of the more immediate importance. "Why" can wait. Of course, this is another way of saying that the teacher must use a psychological approach rather than a logical approach in most cases.

In view of what the various experts tell us, we can try to get significance into our teaching in several ways. Let us constantly remind our pupils that mathematics is organized common sense; and if any part of it does not make sense to them, they should ask us to help them think it through. *We want the theorems of mathematics to make sense in their minds so that they will remember them well and use them as reasons for what they do in the solution of problems.* It is probably best to present a new topic with a short, direct logical approach, if possible, leading the pupils quickly to the theorem and its application to simple exercises. Frequently the best we can do is to relate the topic to what the pupils already know. For example, in the teaching of the theorem that the product of numbers of the same sign is positive and of different sign is negative, a number of pupils naturally want to know why two negative numbers yield a positive product. I tell them that I shall explain the matter n a day or two, and for the time being, they are to do the work as I have told them. If two or three days later I am asked by a pupil to give an explanation, I know that he is alive and sincere. Whether reminded or not, I explain the necessity of this theorem by showing with a numerical example that it makes the distributive law of multiplication hold for negative numbers. I could also cite applications from magnetism and other branches of physics, but I hold with the mathematicians that it is best to base mathematical theorems on mathematics. I mention how beautiful and easy the use of negative numbers makes the solution of problems in optics, electricity, and other fields of science. Of course, we should use applications of mathematics, but we should be remiss if we did not point out and emphasize the mathematics theorems involved.

With regard to the teaching of meaning in mathematics, perhaps it is safe to draw the following conclusions:

1. Drill is essential to skill in mathematics, but drill alone rarely results in complete understanding. Most people seem to understand whole numbers well, but beginning with fractions their ideas are apt to be vague and even illogical.

2. While elementary and secondary mathematics teachers cannot for psychological and educational reasons present their courses with complete logical rigor, they should continually remind their pupils that mathematics is organized common sense and should make every effort to make mathematics appear sensible to them in every particular.

3. Children should be encouraged to grow mentally into the adult interest in the reasons underlying what we do in mathematics. Incidentally, the logic of mathematics is the logic of all reasoning and is the same logic as that used in the solution of problems that are not usually classed as mathematical.

4. No mathematics course, even a so-called practical course, should ever be

taught as a terminal course. The teacher should always feel that he is laying a foundation upon which another teacher or the pupil himself is to build.

5. The teaching of applications in a mathematics course is justifiable only on the grounds that the application of mathematical theorems is being stressed. To teach applications from any other point of view is to encroach upon the fields of the commercial, shop, and science teachers; and if we do that, where in school are the pupils going to learn any mathematics?

6. Mathematics teachers can best serve those who will use mathematics in other fields by teaching them the theorems of mathematics and by making as clear to them as possible the logical structure of mathematics.

Mathematics in the Senior High Schools Differentiated According to Needs

The Second and Third Tracks of Mathematics Courses

By BENJAMIN BRAVERMAN
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THOSE of us who are desirous of having Mathematics make its maximum contribution to the training of every boy and girl for intelligent and happy living will acclaim the recommendation of the Commission on Postwar Plans of The National Council of Teachers of Mathematics, in its first report* that mathematics curricula from now on be organized in three distinct series or tracks or sequences. The first track, which the Commission has labelled *sequential mathematics*, is intended for those planning to pursue further the study of mathematics and the pure and applied sciences based on it. The name *sequential mathematics* may not be the best way to describe this first track. For are not the other two tracks to be sequential too? Perhaps *specialized mathematics* might describe this sequence better since it is intended for pupils who plan to specialize in mathematics. The other two tracks have been labelled by the Commission *related mathematics* for those planning to go into industry and *social mathematics* for the large number of pupils in neither of the already mentioned two categories; that is, pupils interested in neither industry nor college; or if planning to go to college, not intending to pursue studies requiring the expert, specialized knowledge of our subject needed by the pupils in the first category. These are the pupils who suffered most from our mathematical diet in prewar times. For we either let them starve entirely or we gave them acute mental indigestion by feeding them the wrong mathematical diet.

It will be my purpose here to discuss rather sketchily, what has already been done to implement the proposed three tracks of mathematics courses, particularly in our local system, and what still remains to be done. I also propose to touch upon some of the many obstacles that will have to be overcome before we shall be able to bring about a full realization of the three track program.

Since there is no ill wind that does not blow some of us some good, the tragedy of World War II has brought us some good in that it has supplied the impetus for the inauguration in our schools of the second track of mathematics courses, courses designed for those going into industry. The initial course in this track or sequence in our own local schools is called "Related Mathematics." Recently my attention was called to the literature of a similar course in the Los Angeles system where the course is called Military and Industrial Mathematics. It is true that these courses were introduced to serve and for the duration do serve a definite war need—namely, that of giving the mathematical illiterates among our male school population the basic knowledges and skills of arithmetic, algebra, geometry, and trigonometry to enable them to carry on with some assurance of success the pre-induction war courses they may be assigned to while still in school and later the many tasks of a technical nature they may be assigned to in the Armed Forces. It is true, too, that tradition, vested interests, or call it what you will, in some school systems have made necessary certain compromises that have very definitely hindered a reasonable realization of the main objectives of the course. Thus, in our local system, it has been officially ruled that one term of business arithmetic and only one term of the

* See THE MATHEMATICS TEACHER for May 1944.

Related Mathematics course may be considered the equivalent of a year's study of the full course. But already some schools are using the opportunity afforded them by the presence of this course to direct to it pupils who could profit very little by an exposure to the traditional or specialized course on the same grade level in the first track. Also looking at this new course from the broad point of view of education rather than the war emergency, we find that it is helping to brighten up the gloomy mathematical picture that existed in our schools in pre-war times.

What were the main outlines of that picture? You know them as well as I. Mathematical illiterates being turned out by the thousands or tens of thousands; charlatonic general educators gloating over this fact and urging that more and more of our pupils steer clear of any mathematical training; competent school people, deeply concerned over this situation, earnestly pleading for mathematical courses that would help meet the needs, abilities, and interests of the large mass of pupils; our own die-hard traditionalists crying out that mathematics cannot be taught to morons; our forward-looking teachers trying hard to meet the situation by means of modifications of the traditional courses through accretions or deletions of topics and finding that they either had a course much too crowded for proper mental digestion by even the brighter pupil, let alone the non-academic pupil assigned to such modified course, or else that they had such an emasculated course that while doing very little for the non-academic pupil, it was providing excellent training for the other type of pupil in habits of passive, lazy, and slothful thinking and work.

There is neither pleasure nor purpose in recalling this picture except to point out that we as educators first, and as mathematics teachers second, must see to it that we do not return to a similar situation in the post-war period.

We must, therefore, hold on tenaciously to the gains already made by the inauguration of a two-track plan of mathematics courses as a result of the war. Spurred on by this start, forward-looking teachers in our local system, and I do not doubt that the same thing is true in other systems, have planned and have succeeded in getting the authorities to approve other courses that now provide three years of a continuous, sequential exposure in the second track of mathematics related to industry. I am also happy to say that in a few of our local schools these courses covering at least $2\frac{1}{2}$ years of this sequence are actually in operation at the present moment. I have prepared mimeographed copies of the main objectives and content of these courses as well as of the Related Mathematics course, which has served and should continue to serve as the initial course for the second track.

Even a cursory examination of these outlines will quickly reveal what is lacking in our present local set-up to fully implement the recommendation of the Policy Commission for a three-track series of courses. We have nothing in the ninth year as the basic course in the third track of mathematics courses, mathematics related to social and economic situations. For want of such a much-needed course, junior high school administrators in this city are either going back to the doubtful practice of eliminating from mathematics study those who cannot handle the regular ninth year course or else are assigning them to business or shop arithmetic.

I think most of us are agreed as to what the chief ingredients of a ninth year course in social mathematics should be. We all feel, I believe, that such a course should include, on the practical side, such topics as budgeting, insurance, installment buying, thrift, and taxation, through which the pupil will learn to solve those problems directly affecting the welfare of himself and his family; on the intellectual side, such a course should include such topics as the index number, the interpretation of graphs, and the elements of statistics,

through which the pupil will learn to make intelligent decisions as a citizen. Finally, on the appreciative side such a course should contain such topics as the history and properties of our number system, indirect measurement, certain of the aesthetic aspects of mathematics such as symmetry, and the historical development of the subject, topics through which the pupil will acquire a respect for mathematics because of its important contributions to the progress of civilization and a tolerant attitude toward other races and peoples because of their varied contributions to its development. How different such a course sounds from the traditional course or from such substitutes as business arithmetic that we have been feeding our pupils in the third category.

The question must have arisen in your minds by now as to when the three-track plan is to begin to operate. I hope I have not been misunderstood as advocating the three-track plan as late as the beginning of the ninth year. I am sure that we all agree that these three distinct tracks should begin to operate at that point in our school organization when it becomes evident through our knowledge of the pupils' needs, interests, and abilities that one track rather than another will provide a pupil with the most effective educational experiences.

I mention this now because I should like to say a few words with regard to the instructional material and methodology that must be employed to make our teaching effective in these second and third tracks. Not that these points do not apply just as well to effective teaching in the first track, but failure to observe the points in the first track may not be accompanied by such disastrous consequences as such failure in the second and third tracks.

First, the problem material used must sell itself to the pupil because of its vital connection with life situations that already are part of the pupil's experiences or soon will be. The large number of formal examples designed to establish mathematical skills for their own sake, so typical of the teaching of traditional mathematics in the past, must give way in the second and third tracks to examples clothed in concrete practical terms that will appeal to the craving of the young adolescent for the useful. Perhaps one or two illustrations will make this point clearer. Thus the example: *Change 83/154 to a decimal fraction to the nearest thousandth,* should give way to the example: *At the close of the 1944 baseball season, the New York Yankees had won 83 games out of the 154 games played. What was its standing?* It is assumed, of course, that the word *standing* has been previously explained by the teacher. Also, the example: *Draw an angle of 160° with your protractor, should give way to the example: A destroyer sights an enemy submarine bearing 160° from the position of the destroyer. If D is the position of the destroyer at the time of observation, draw a line representing the direction of the submarine.* Here again, it is assumed that the teacher has informed his pupils of the two arbitrary conventions used in navigation for determining an angle.

Not only must we make the problem situation vital in the second and third tracks, but we must exploit such situation to the fullest degree in obtaining for the pupil a maximum of worthwhile educational experiences. Let me make myself clear by giving two illustrations. Thus, instead of telling a pupil that a room measures 15 ft. by 22 ft., and then asking him to draw it to a scale of $\frac{1}{4}'' = 1$ ft., we should give him a mechanic's rule, ask him to make the necessary measurements of the room he is sitting in himself, and then have him select an appropriate scale that will enable him to draw a floor plan of the room in a convenient size on an ordinary sheet of paper. Or instead of giving the pupil the traditional problem of finding the area of a circle given its radius, we should bring a small length of pipe to the classroom, show him what we mean by the term *cross-section,* and then have him

make the necessary measurements to enable him to find the area of the cross-section of the pipe. By this procedure, we would be utilizing our subject not only to develop mathematical concepts and computational skill, but also to train the pupil in the equally important skills of making accurate measurements and drawings. I know that there are some traditionalists among us who claim that these desirable goals are not the concern of teachers of mathematics, that they properly belong to shop and drawing teachers. But those who look upon our work primarily as trainers of youth will think otherwise.

Finally, in these second and third tracks, there must be a determined effort to teach concept and understanding. It is important to do this, of course, in the first track too. But the type of pupil who is exposed to the first track may learn the concept in spite of faulty teaching. Not so with the pupils in the second and third tracks. Questionable algorisms, procedures, short cuts, which have grown up over these years because of our mistaken idea that memory rather than understanding is at the basis of all effective learning must go by the board.

What now are some of these pedagogic barnacles that will have to be discarded? Well, one that I have no hesitation at all in recommending for such discard is the algorism for the addition or subtraction of common fractions, the so-called step ladder form. You are all familiar with it, the one where the lowest common denominator is written once above a horizontal line at the top of the solution and only the numerators of the equivalent fractions are written below.

A questionable procedure that should be abandoned is the mechanical process for the extraction of square root. And of the short cuts that we would be better off without are the mechanical rule for the subtraction of signed numbers and the transposition of terms in the solution of an equation.

Finally, we must not be too impatient to introduce desirable short cuts before the pupil has mastered the fundamental concept and is ready for it. Thus, I should like to see pupils who have difficulty in mastering the concepts of area and of volume, spend perhaps an entire term or an entire year, obtaining areas of various figures by actually counting squares and obtaining volumes of various figures by actually counting cubes before we give them any of the rules for obtaining areas or volumes indirectly. Such an approach may prevent us from covering so much ground as we have in the past, but we would have the satisfaction of knowing that such pupils understand these important concepts.

Before concluding, I should like to say something about some of the obstacles that stand in the way of realizing the three track plan of mathematical instruction. There are two main difficulties, as I see it. Once was touched upon by the Policy Commission in its preliminary report when it said that the second and third tracks must be presented without stigmatizing any group of pupils. The other difficulty is the difficulty attending any pioneer effort.

Several conditions will have to be remedied to remove the stigma from the pupils in the second and third tracks. For one thing, the colleges or those in the colleges responsible for admission requirements will have to realize that a pupil interested in such professions as journalism, law, accounting, or the ministry, does not necessarily need the first track of mathematics courses for successful study for any of the professions mentioned. As a matter of fact, the *third* track would be a much more desirable track for many pupils choosing these professions. Even a pupil who has been exposed to the second track and then decides to go to a technical school should not be denied admission. For if such a pupil has missed some of the manipulative and theoretical parts of traditional mathematics, he has more than compensated for his omissions by a better understanding of what he has been taught.

It should, therefore, not be very difficult for such a pupil, because of his ambition, to make up in the technical school the topics that he missed in the secondary school because of his exposure to the second track.

Then too in our local schools at least, the courses in the second and third tracks must be placed on an equality with the traditional courses in so far as credit for the award of the academic diploma is concerned. The traditional courses have, as you know, extramural examinations, the Regents examinations. In our local schools certain credits must be earned by the pupil as a result of passing these extramural examinations before he can be awarded the academic diploma. Far be it from me to urge similar extramural examinations in the courses of the second and third tracks. But I do believe that we should be thinking of the desirability of instituting local examinations in these tracks which would carry the same credits for the award of the academic diploma as the present Regents examinations in the first track.

Finally, much publicity will have to be done with administrators guidance counsellors, and parents to acquaint them with the objectives and content of the courses in the second and third tracks. Having been exposed themselves to the traditional mathematics of the first track, it is but natural for many of them to view with suspicion and lack of sympathy the courses in the second and third tracks about which they know little or nothing. Parents in particular are inclined to regard the courses in the second and third tracks as definitely inferior to the traditional courses. A good deal of the right kind of propaganda on our part will, therefore, be necessary to change this prejudice on the part of parents and guidance people.

But there is another and even greater obstacle in implementing the three track plan. Teachers who have the desire and ability to develop the courses in the second and third tracks require much time and academic calm to do so. Unfortunately, in a large system like that of New York City, this is not recognized. It is thought that a teacher's day is at least 48 hours, so that after six hours of trying and exhaustive work in actual classroom duties, the teacher can devote another few hours to the labor of preparing the syllabi and content material of new courses of study. Perhaps in some of the smaller systems there is a more humane attitude toward teachers.

Finally, even if teachers do find the time and energy, somehow, to develop these courses, there is this other great obstacle. How can the instructional material they develop be made available to the pupil? Publishing concerns are not willing to take the economic risk connected with publishing something different from the traditional. One cannot blame them for this attitude. School systems, on the other hand, are not inclined to undertake the printing of instructional material for new courses because they do not want to enter into competition with commercial publishing concerns. And so, between the Scylla of economic risk and the Charybdis of no competition between school systems and publishing concerns, little or no progress can be made. This is a serious situation and requires the attention of a national body like our National Council. Perhaps one solution would be to get some educational foundation like the Carnegie or Rockefeller foundations to contribute the necessary funds to carry on a worthwhile project like the implementation of a three-track sequence in mathematics in the schools.

By what means or how soon the obstacles I have mentioned will be removed, it is difficult to say. But we may express the hope that these obstacles will be solved and solved in the near future; because we shall neither have happy pupils nor happy teachers; neither shall we achieve functional competence in mathematics unless we develop effective second and third tracks in mathematics, as well as an effective first track.

Some Thoughts on Tomorrow's Mathematics

By S. L. Berman
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How times have changed! A number of years ago, the educator who espoused the cause of increased mathematical study for secondary school pupils would have been tolerated in some quarters, considered eccentric in others, but would have been ignored completely or not too quietly ridiculed in most educational circles. Now, not only are schoolmen deeply interested in the extension of mathematical education, but their concern is not limited to related mathematics or to social mathematics. It has been rediscovered that there *is* a place in the high school curriculum for the traditional sequential courses in mathematics, a place of importance in the world of tomorrow.

We shall not dwell upon the reasons for the increase which has taken place in mathematical study. They are well known. It must be our purpose now to make certain that these gains are consolidated and held against the attack which will be attempted by general educators who are strangely silent at present. We must raise the general level of mathematical literacy. We must insure functional competency in mathematics. Primarily, however, we can best prepare ourselves for the coming educational battles by unifying our concepts as to what constitutes an acceptable level of literacy and functional competence in mathematics and by implementing such a program.

Let us define the field. According to the First Report of the Commission on Postwar Plans of the National Council of Teachers of Mathematics,* sequential courses (in mathematics) means the four years of work (grades 9–12 inclusive) providing rigor and continuity and to be obtained by greatly improving the traditional sequence: algebra, geometry, and trigonometry. We should not confine these courses to those students who will enter the technological professions. It is a mistake to exclude from these courses pupils who want to obtain worthwhile basic disciplines in analysis, scientific method, and rigorous thinking, all of which are vital to the good lawyer, doctor, economist, or business administrator. In recent years, most non-technological professions have undergone change because of the introduction and extension of mathematical treatment of their fields. Let us rather apply the general principle; namely, "that any course be open to any youngster who can continue to show that it is profitable to him." In making the recommendations, we shall follow the suggestion in the "First Report" that the sequential courses can be greatly improved by changes in each of the three factors involved: the pupil, the material, and the teacher.

The pupils who will take this traditional sequence in mathematics need to be selected carefully. We do not mean to imply that such segregating processes be conducted on a basis of separating the wheat from the chaff. We must not make the mistake of classifying these youngsters as superior to, or in any way better than, those who will take the courses on other tracks. It is completely democratic to realize that all kinds of mentalities compose the human race. None of us is better than his fellows, but we must recognize that we are all differently constituted, motivated, activated, and satisfied. We should endeavor to remove that peculiar dignity which attaches itself to an ability to master academic processes. We must give both child and parent the feeling that we are trying to match the peg to the hole.

For years, our elementary schools and junior high schools have been priding themselves—and telling the world, too—that

* See The Mathematics Teacher for May 1944.

they keep complete records of their children. Reading tests, general intelligence tests, anecdotal records, guidance interviews, records of class and extraclass activities, home visitations, all these and others should furnish a complete inventory of the child's abilities and inclinations. We cannot but feel that the cumulative record of eight or nine years can indicate the pattern of the pupil's further education. We submit that high school guidance has not fully availed itself of those existing accounts. Instead, one section of the record, say the I.Q. and the subject grades, or any other combination, but not the entire record, has become the determining factor in deciding the nature of the youngster's high school educational level. We have created a vocational pupil, a non-academically minded pupil, a slow pupil, but in most cases, a maladjusted pupil. All our further efforts in the direction have not compensated for the initial maladjustment.

To take this record as a final basis for judgment is poor mathematical procedure. We need verification. For checking purposes then, a series of tests, interviews, consultations, call them what you will, should be instituted before the pupil's future educational path is indicated. Properly, that is the function of guidance men and testing experts acting in conjunction. We must assume that such a series can be devised which will make for a reliable predictability quotient. However, to bring into our thinking the idea that at least a better means of prognostication can be determined requires no great stretch of the imagination.

We cannot hope to predict with scientific accuracy. We must expect that in some cases, the record will paint a distorted picture of the child's abilities, interests, and attitudes. Provision for switching from one track to another must therefore be made. We must be ready to admit mistakes and to rectify them as soon as possible. We must include among our failures not only the actual failures but also those who have failed to live up to their capabilities. Care must be taken to distinguish between a merely satisfactory minimum achievement on one level and a larger measure of success on another. The attainment of a passing grade should not be construed as denoting success if the pupil is capable of doing superior work along other lines. Isn't it the aim of public education "to promote the general welfare by encouraging and developing in each individual his best personal and social competence?"

Substantial changes should be made in the material of the traditional sequence in mathematics. For a long time, the argument has been put forth that the secondary school can increase the effectiveness of mathematical instruction by the introduction of illustrative materials and practical problems. We must recognize that such practical applications require a technical setting, i.e., some technical apparatus but not necessarily a technical background. Without this equipment, without the opportunity to see mathematics at work, our pupils have been subjected to practical problems which not only remain academic to them, but in many cases defeat their own purpose by being anything but practical or scientific. Thus, in some well-known textbooks, it was found that the authors thought that an automobile radiator had a capacity of 20 gallons; that an ordinary dry cell could develop 40 amperes in a problem; that an oar in an oarlock of a rowboat was a first class lever (without appreciating the fact that the water, not the oarlock, was the fulcrum); that a surveyor never bothered with the height of his instrument; that the dividend from stocks was determined by taking a per cent of the cost; that it was not necessary to refer to the molecular weights or to the chemical equations in giving a problem about chemical reaction; and that it was quite an ordinary practice to measure the height of a tree when the angle of elevation of the sun was 3°. Are not such so-called practical problems ridiculous? Still

most textbooks contain some applications which are far removed from reality.

How much better it would be were we to teach direct variation by a mathematical treatment of the data obtained by performing a simple experiment in Hooke's Law! How much more meaningful would learning become for the student who sees inverse variation at work in a demonstration of belted pulley speeds in the shop, or with a simple lever system! An angle of elevation would be something less abstract to the boy or girl who used, say, a hypsometer to find one. The sine curve would assume its proper importance in our teaching were we to produce it by a simple motion or were we to correlate its characteristics with the teaching of the alternating current in the physics class.

That word "correlate" should stir pleasant memories and at the same time bring to mind its partner, "integrate." There are many reasons to which we can attribute our failure to realize fully the aims implied in those two words. We, in the field of mathematics are guilty of having maintained an attitude of high-and-mighty aloofness with respect to contributions which other fields may have had to offer. We have considered mathematics as *the* important subject in education instead of *an* important area. We have been hesitant in descending—*descending*, mind you—to the level of the practical, preferring to keep mathematics on an academic plane. We have continued to include in our syllabi material which has long ceased to provide any meaningful or useful experiences for our pupils. Honestly now, do we really believe that a pupil is vitally affected by the discovery that 8 years ago, his father was 10 times as old as he was, or that a purse contains 18 nickels and 23 dimes?

Administration, including supervision here, has not shown the willingness and the courage needed to make changes. There has been a "do-the-best-you-can-under-the-circumstances attitude" with weak attempts to remedy those circumstances. Some two years ago, a bulletin of the New York State Department of Education said, "As to war problems, the department is not yet ready to recommend a wholesale injection of such material into our courses. We should be very sure that we do not get into an amateurish and futile dabbling with all sorts of technical material." One cannot deny that a "wholesale injection" of such material will create a disproportionate emphasis on applications and may cause a sacrifice of attention to fundamentals. One must decry, however, the extreme caution expressed by the department. How are we ever going to "be very sure" about things without getting to work on the problem?

Criticism is too easy! We all know that there are many things in our present setup which need modification. Let us proceed to the business of making recommendations. In an article entitled, "Postwar Secondary School Education," which appeared in the January 1945 issue of *High Points*, I suggested a reorganization of the secondary school schedule which, by removing the "Carnegie Unit," would provide for a prescribed three-year sequence in mathematics for every pupil. At the same time it would provide for required sequences in other subject areas. With such a plan, the pupil's program would be more regular and definite. We could be more certain of maintaining our hold on the student for at least three consecutive years. Not only could continuity of learning and integration become attainable, but also correlation would increase to be a meaningless term. Cooperation in horizontal planning to the end that materials and procedures in any given time unit of instruction become mutually complementary, should be the concern of supervisors of *all* departments of instruction. Although complete correlation may not be realized, no one will deny that it is possible to devise units, common to more than one department, which would receive varying treatments. It must be understood that one core curriculum is not being suggested here but rather many such themes or cen-

ters of interest or emphasis. For example, on a given day, the same topic might be studied in both the mathematics and the art classes, or on another day, the mathematics and science departments would plan their instruction together; on still another day, the same scheme would find the mathematics and shop teachers participating. The number of "correlation days" per week, or per term, would depend upon the number of such units it would be feasible to attempt. We do not plan to change our teaching except in this direction. Formal instruction would still be given, in fact, the methodology need not be changed on *this* account. However, the task of devising such units requires the combined efforts of many supervisors.

The successful introduction of such a proposition requires that we establish also a set of corollaries. It would be advisable that mathematics and science be scheduled in consecutive periods so that double periods in both mathematics and science could be arranged more easily when desirable. Also, it may be necessary to alter the physical characteristics of the mathematics classroom. A demonstration table in addition to various demonstration devices, equipment, and instruments, all need to become part of the specialized mathematics room. Furthermore, the course of study should include some provision for the pupil to use his hands in some sort of handicraft, call it "mathcraft" if you will, such as drawing, designing models, making models, or other manual arts. Realities should be brought home to the pupil.

Vertically, also, changes should be made in the materials of mathematics instruction. Because the proposed pupil schedule guarantees that we will have the pupil for at least three consecutive years, we can now construct a three-year course of study, rather than retain the three one-year courses. Much of the present overlap of courses could be eliminated and greater continuity could be achieved. Such a three-year unit must break down the compartmental sections which still exist. The total unit should consist of arithmetic, algebra, intuitive and mensurational plane and solid geometry, graphic representation, direct and indirect measurement including trigonometry, scale drawing, map making, consumer education, etc. to the almost total exclusion of demonstrative geometry and the more theoretical aspects of mathematics. Only fundamentals should be taken and these should emphasize applications to problems in the corresponding science and other concurrent courses, and to *real* problems occurring in the professions or in industry. A wealth of such problems could be obtained through investigation of the industries themselves or through governmental agencies. It is not intended that the course be popularized or toned down but, in general, the material to be included would have to stand a pragmatic test. We should teach material not for what we consider the beauty of the theory or for the reason that we have *always* taught it, but primarily for the use to which this material can be put. We must be able to show the pupil that the study of mathematics will bring him if not immediate value, then benefits which will not be postponed too long.

These courses of study in the pre-senior years can be so differentiated that a minimum of educational waste resulting from failure should be realized. On the three proposed tracks, differentiation should be in the directions of content and time allotment. The traditional courses should contain both a greater number and a larger ratio of scientific and technical applications with correspondingly less time allowed for the completion of each topic. Although some demonstrative aspects of mathematics should be covered, they should be consigned to a place of lesser importance in the whole scheme. Good usage rather than the formal grammar of mathematics should produce both a desire and an ability on the part of the youngster to develop the habit of clear exact speech as a prerequisite or corequi-

site to preciseness of thought. Rigor and fundamental principles need not be considered lost by the adoption of such content and method. College requirements also need not be sacrificed.

Many will wish to continue with a fourth year of mathematics. Here, work more theoretical in nature, or more specialized, may be offered. Demonstrative geometry, logic, advanced trigonometry and algebra, or enginering mathematics, surveying, navigation, should properly be included. The emphasis should shift from practical to theoretical, from terminal to propaedeutic, from general to special. By this time, proper guidance will have indicated the direction of his future educational life to the student, if not his professional life.

Before we come to the discussion of teaching qualifications, we must make some hopeful assumptions. Teacher remuneration will be such that financial worries of the profession will cease to exist. Just as a physician or an accountant is enabled to devote his full time to the practice of his profession, so should it be made easy for a teacher to spend his life in educational activity. A teacher should not have to write a book, or to run a camp because he needs the extra income. Engaging in extraclass activities and in self-improvement activities should be part of his growth in his professional life. It is however a curious fact that certain educators are opposed to contemplated state or federal legislation which would ease the burden of education in general and of the teacher in particular.

The adoption of the suggested plan for the traditional mathematics requires that a great change take place in the selection of teachers. Teachers need to be selected on a basis of variety of interests and abilities. The teacher's background and outlook must be broad enough to allow him to apply contributions from fields other than his specialty to the education of the child. The teacher must accept some means of developing a more professional attitude so that he can allow himself to be kept alive to changes that are taking place and adapt himself and his methods to these developing needs. Finally the teacher should be required to demonstrate a desire and ability to live with boys and girls.

Our previous standards for teacher training and certification are inadequate for the selection of men and women who will teach tomorrow's mathematics. Scholarship in mathematics should not be the determining factor. It is not too much to expect of the teacher of tomorrow that he be familiar with the applications of mathematics to other fields, such as the mechanic arts, engineering, aviation, economics, statistics, and even the fine arts. The new mathematics demands such versatility. Training for the teacher of mathematics should include special courses in methods of correlating mathematics with other subjects. The mathematics teacher of the future should have some training in a mathematical laboratory where he will learn how to use instruments and machines as well as his hands. Some steps in that direction have already been taken in New York City, although not for the mathematics teacher. Science licenses have been combined; history and economics are required for a license to teach the social sciences; a supervisory position in English and social studies in the vocational schools has been created. Good, but how about the mathematics teacher?

All these recommendations for the traditional sequential mathematics of tomorrow have been given in more or less general terms. To go into detail would, in some instances, require a treatment of book length. In other cases, the suggestions hinge upon aspects of our educational problem which it is not within our province to discuss fully. However, an attempt has been made to indicate the direction in which we must use our best efforts. The saying goes, "Direction first, distance later."

The Rhumb Line on the Sphere

By Mildred Hopkins
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Pupils are sometimes interested in computing and comparing the great circle and the rhumb line distances between two arbitrary points on the earth's surface.

By using the formulas of spherical trigonometry, the great circle distance can be found. In order to derive a formula for rhumb line distance, one can make use of spherical trigonometry formulas and of calculus processes.

The rhumb line is the loxodromic curve on the earth's surface which cuts all meridians at a constant angle. This angle is determined for a particular loxodrome when the latitude and longitude of two points on the curve are known. One travels in a constant direction when following the path of this curve. For short distances, we sometimes think of it as following the path of a straight line. Due to the curvature of the earth, however, we are actually tracing out a curve when we travel in a constant direction.

In order to obtain the equation of the rhumb line, consider the earth as a sphere. Let r represent the radius of the sphere, u a circle of latitude, v a meridian, and α the angle made by the loxodrome with two arbitrary meridians. Let u be measured from the north pole. Then.

$$\tan \alpha = \frac{r \sin u \, dv}{r \, du} = \sin u \frac{dv}{du}.$$

Then

$$(1) \qquad \frac{dv}{du} = \tan \alpha \csc u.$$

By integration, $v = \tan \alpha \log_e \tan \frac{u}{2} + C_1$.

Since α is determined by the latitude and longitude of the departure and destination, u_1, v_1 and u_2, v_2, then

$$(2) \qquad \tan \alpha = \frac{v_1 - v_2}{\log_e \tan \frac{u_1}{2} - \log_e \tan \frac{u_2}{2}}.$$

The distance s along the rumb line between two arbitrary points on the sphere can be represented by substituting the expression for $\frac{dv}{du}$ of equation (1) in

$$ds = r\sqrt{1 + \sin^2 u\left(\frac{dv}{du}\right)^2}\, du,$$

the length of an arbitrary curve on a sphere.

This substitution gives, $ds = r \sec \alpha \, du$

(3) By integration, $s = ru \sec \alpha + C_2$.

When $\sec \alpha$ is expressed from equation (2) and substituted in (3), we get

$$s = r(u_1 - u_2)\sqrt{1 + \left(\frac{v_1 - v_2}{\log_e \tan \frac{u_1}{2} - \log_e \tan \frac{u_2}{2}}\right)^2}.$$

Since one minute of distance along a great circle of the earth's sphere represents one nautical mile, then the circumference of the earth is $360(60) = 21{,}600$ nautical miles.

Since $C = 2\pi r = 21{,}600$, then

$$r=\frac{10,800}{\pi} \text{ nautical miles.}$$

$$(4) \qquad s=\frac{10,800}{\pi}(u_1-u_2)\sqrt{1+\left(\frac{v_1-v_2}{\log_e \tan\frac{u_1}{2}-\log_e \tan\frac{u_2}{2}}\right)^2}.$$

One should keep in mind when using formula (4) for rhumb line distance that u_1, v_1 and u_2, v_2 are in radian measure and that u_1 and u_2 are measured from the north pole. If one wishes a formula where degrees and minutes and logarithms of trigonometric functions to the base 10 can be used, then formula (4) becomes.

$$(5) \qquad s=60(u_1-u_2)\sqrt{1+\left(\frac{\pi(v_1-v_2)}{180(2.303)\left(\log_{10} \tan\frac{u_1}{2}-\log_{10} \tan\frac{u_2}{2}\right)}\right)^2}.$$

In solid geometry, we learn that the minor arc of a great circle is the shortest distance between two points on a sphere. Pupils seem pleased to obtain this information again by means of formula (5) and the great circle distance formula.

By using formula (5), we find that the rhumb line distance from New York to London via Newfoundland is less than the rhumb line distance direct from New York to London. Thus the distance along two or more rhumb lines, properly chosen, may be less than that along one rhumb line.

By use of formula (5), we see why navigators fly a series of rhumb line chords (loxodromic arcs) approximating a great circle path. For short distances, the rhumb line distance is approximately the same as for the great circle distance.

In solid geometry, we learn that a spherical triangle which has great circle arcs as sides always has the sum of two sides greater than the third side. It is interesting to note that this does not seem to hold when the three sides of the triangle (not spherical) are loxodromic arcs.

Formula (5) reduces to

$$s=60(u_1-u_2),$$

when the two points have the same longitude. The result is the same as the great circle distance since, in this case, the rhumb line is a great circle passing through both poles of the earth's sphere.

Ordinarily, the rhumb line on a sphere is a spiral which approaches both poles as limits. Its total length is $\pi r \sec \alpha$, when u varies from 0° to 180°.

Notice

In the January, 1945 issue of THE MATHEMATICS TEACHER, it was announced that the posters made by students for the Women's Mathematics Club of Chicago had been turned over to the Public Relations Committee of the National Council of Teachers of Mathematics, and would be available for borrowing if the borrower guaranteed to pay the express on the posters.

Since the announcement, many schools have used the posters for exhibits, sectional meetings, etc. and were delighted with them. The posters will again be available this year, if the express expenses are paid by the borrowers. Request should be made to the new chairman of the Public Relations Committee, Miss Veryl Schult, Wilson Teachers College, Washington 9, D. C.

The Next Step in Planning for Post-War Mathematics

By James H. Zant
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for
The Commission of Post-War Plans

The general response to the Commission's Second Report[1] has been amazingly good. Numerous letters of commendation have already been received by members of the Commission and the report has already been discussed critically by several summer groups like the Institute for Teachers of Mathematics at Duke University. We hope other groups will study the report and submit constructive criticisms.

The Board of Directors of the Council are agreed that the next job of the Commission "is to create, or cause to be written, a relatively small but effective pamphlet on Mathematics to be used in the guidance of the junior and senior high school pupil. Its purpose will be to make clear what mathematics has to offer. The idea is that it be directed to the student but widely distributed to mathematics teachers, who in turn may transmit the pamphlet or at least the ideas, to home room teachers and other persons with responsibilities for guidance."

At a meeting of the Commission in New York on July 1 and 2, which was unofficial, since the majority of the members could not come, the proposed guidance pamphlet was discussed. It was proposed that as a first step the Commission set up a list of topics dealing with guidance for mathematics students. Such a tentative list appears below. This list will be further discussed and revised at the next regular meeting of the Commission in October. It is then proposed that members of the Commission attempt to obtain an expression of opinion and definite facts concerning these topics.

This will require the cooperation and assistance of individual teachers, city study groups, workshop and summer school groups and any others who are willing to help. It is the desire of the Commission to make this data thoroughly reliable and have it actually answer the questions which arise constantly for pupils and teachers in the secondary school. To do this we will need the help of the teachers in these schools as well as a fresh practical approach to the problem of guidance from the standpoint of the pupil and his future job. Hence this is a plea to all forward looking teachers of mathematics to offer their help, either as individuals or as groups, in solving this important problem. If you will contact the writer of this article, Professor Schorling or any member of the Commission, your help will be gladly received.

A tentative list of topics dealing with guidance is given below.

1. Obtain statements from key men in industry, business, the Armed Forces, etc., showing their opinion of the need of mathematics for people under them. [These statements should include something of shortages observed, emergency training classes, etc.]
2. Determine the mathematics needed by government officials and Civil Service employees.
3. Determine the mathematical training a person must have after grade 8 in order to qualify for certain key professions, as, for example, the engineer, the navigator, the certified public accountant, the actuary, etc.
4. Determine the mathematical knowledge the citizen needs to read pa-

[1] See Mathematics Teacher, May 1945.

pers, magazines and the simplest government bulletins.

5. Determine the predictive value of success in mathematics courses in regard to various occupations and in the Armed Forces.
6. Determine the mathematical knowledge needed for studying courses in business education and for simple business jobs.
7. Determine the actual demand for well-trained mathematicians in the various ramifications of modern life. [Distinguish between the mathematical specialist and the engineer or physicist who merely uses parts of elementary mathematics a great many times.]
8. Determine the special types of problems involved in guidance of mathematics students at the Junior College level.
9. Give certain illustrations showing how key concepts of mathematics carry through and are used in many different settings in life situations.
10. Determine the use that can be made of girls in the field of mathematics.
11. Make a list of questions pertaining to the guidance of students of mathematics and show where the answers appear in the pamphlet.

Other topics will probably be added as time goes on. Almost certainly these will be revised and clarified by further discussion of the Commission and criticism from without. The young people in our secondary schools and junior colleges have a right to know the opportunities which will be available to them if they successfully complete certain courses in mathematics. They have a right to know the mathematical requirements they must meet if they expect to enter and succeed in certain professions. These facts and many others are not available to those in charge of guiding our youth or even to those who teach the mathematics courses. The Commission hopes with your help to do something about it.

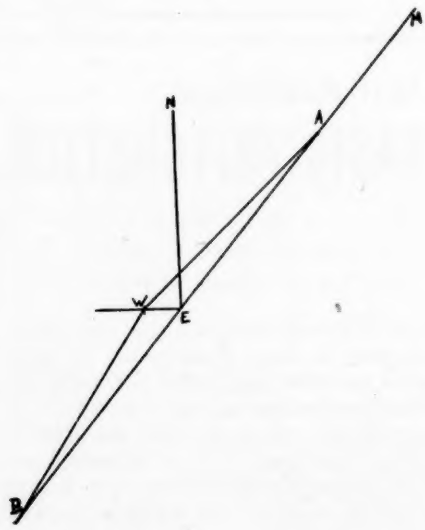
◆ THE ART OF TEACHING ◆

Mathematics and Radius of Action

By John A. Tierney
Plainville, Conn.

There are several methods of solving a radius of action problem. An interesting exercise in high school mathematics is to demonstrate that the result is the same whichever method is used. Let us consider the following problem:

A plane with airspeed 200 mi. per hour starts out on a track of 40°. If the plane has fuel for one hour's flying time (plus safety margin) and the wind is blowing 30 mi. per hour from 90°, how far can the plane fly and still return to its base?



Method I

Draw the wind vector EW and the track EM. With center W and radius 200 draw arcs cutting EM at A and EM extended at B.

$$EA = 179 \text{ m.p.h.} = \text{the ground speed out}$$

$$EB = 218 \text{ m.p.h.} = \text{the ground speed in}$$

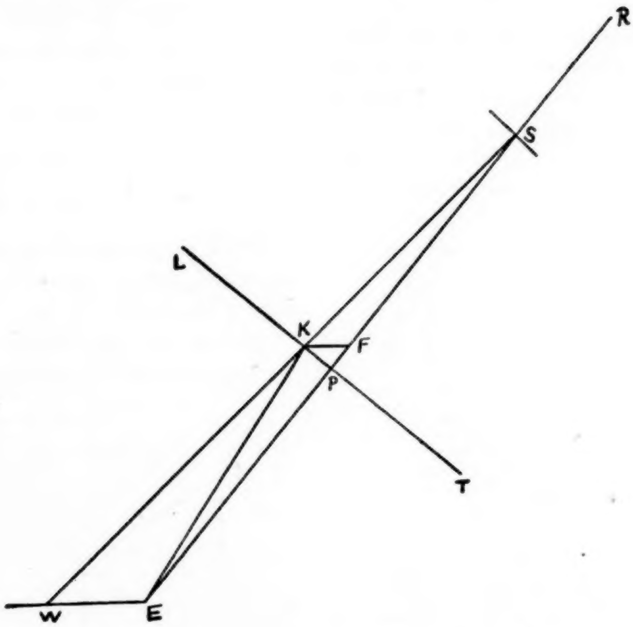
Using the radius of action formula which can be derived as a time-rate-distance problem in first year algebra:

$$\text{R.A.} = \frac{(\text{g.s.o.}) \times (\text{g.s.i.})}{(\text{g.s.o.}) + (\text{g.s.i.})}$$

$$= \frac{(179)(218)}{(179) + (218)} = 98 \text{ miles.}$$

Method II

Draw the wind vector EW and the track out ER. With center W and radius 200 draw an arc cutting ER at S. Construct LT the perpendicular bisector of ES and call its intersection with WS point K and with ES point P. Through K construct a line parallel to EW cutting ES at F. Then EF is the radius of action. WK is the heading out and KE the heading back.



Now to show that EF is the same result obtained by the formula in Method I let $EP=a$, $PF=b$, and $FS=c$.

Since EWS is a one hour wind triangle ES or $(a+b+c)$ is the ground speed out.

KFE is a wind triangle for the return trip but is not based on one hour.

Since triangle EWS is similar to triangle SFK,

$$\frac{WE}{KF}=\frac{SE}{SF} \quad \text{or} \quad \frac{(a+b+c)}{c}.$$

Now, since KF is the wind vector for the return trip and

$$WE=\frac{(a+b+c)}{c}\times\overline{KF},$$

the ground speed in must equal

$$\frac{(a+b+c)}{c}\times\overline{EF} \quad \text{or} \quad \left(\frac{a+b+c}{c}\right)(a+b).$$

In other words, to change triangle EKF to a one hour triangle each side would have to be multiplied by

$$\frac{(a+b+c)}{c}.$$

Substituting these values in the formula of Method I we have:

$$\text{R.A.}=\frac{(\text{g.s.o.})\times(\text{g.s.i.})}{(\text{g.s.o.})+(\text{g.s.i.})}$$

$$=\frac{(a+b+c)\left(\frac{a+b+c}{c}\right)(a+b)}{(a+b+c)+\left(\frac{a+b+c}{c}\right)(a+b)}.$$

Since P is the mid-point of ES, $c+b=a$ or $c=a-b$.

Substituting $(a-b)$ for c:

$$\text{R.A.}=\frac{(2a)\left(\frac{2a}{a-b}\right)(a+b)}{(2a)+\left(\frac{2a}{a-b}\right)(a+b)}$$

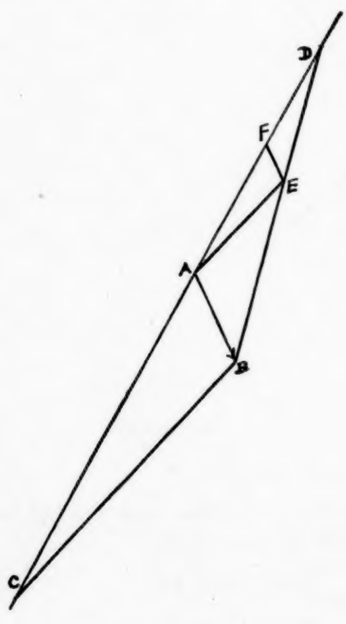
$$=\frac{4a^2(a+b)}{2a(a-b)+2a(a+b)}$$

$$=\frac{4a^2(a+b)}{(2a)(2a)}=a+b.$$

But $a+b=EP+PF=EF$, the value taken for the radius of action in Method II.

In this problem the available flying time was taken as one hour; if the time is not one hour multiply the radius of action for one hour by the number of hours.

Another method for finding the radius of action graphically is given below.



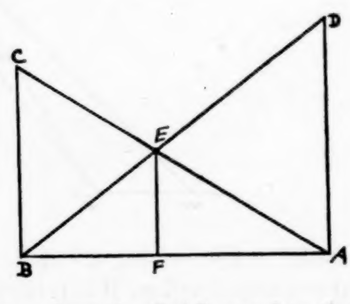
AB = the wind vector
$BD=BC$ = the air speed
AD = the course out
AC = the course back
Draw: $AE\|BC$; $EF\|AB$
Then: AD = the ground speed out
AC = the ground speed back
FA = the R.A. (for 1 hour)
BD = the heading out
BC = the heading back
$\angle BDA$ = the wind correction angle

It is easy to show by plane geometry that

$$AF=\frac{AD\times AC}{AD+AC}.$$

If the two ground speeds are known the radius of action can be found by the following diagram.

Choose any two points B and A on a straight line. Erect perpendiculars at these



points and mark off AD equal to the g.s.o. and BC the g.s.i. Draw CA and BD. The perpendicular from their intersection E to AB is the R.A.

The proof is left to the reader.

IN OTHER PERIODICALS

By NATHAN LAZAR
Midwood High School, Brooklyn 10, N. Y.

The American Mathematical Monthly
February 1945, Vol. 52, No. 2

1. Adler, Claire Fisher, "An Isoperimetric Problem with an Inequality," pp. 59–69.
2. Hamming, R. W., "Convergent Monotone Series," pp. 60–72.
3. Thomas, T. Y., "Maximum Angular Variation under Small Displacements," pp. 73–81.
4. Craig, H. V., "A Matter of Motivation," pp. 82.
5. Larsen, H. D., "On the Calculation of Bond Yields," pp. 83–86.
6. Stewart, B. M., "Two Rectangles in a Quarter-Circle," pp. 92–94.
7. War Information: The ESMWT program of Pennsylvania State College. The Navy Campaign for Radar Technicians. The Educational Program for Canadian Veterans.

National Mathematics Magazine
January 1945, Vol. 19, No. 4

1. Sanders, S. T., "An Important Diagnosis."
2. Reynolds, Joseph B., "The Mathematics of a Nut Cutter," pp. 159–162.
3. Fort, Tomlinson, "Taylor's Formula and Sterling's Numbers," pp. 163–170.
4. Funkenbusch, William, "Maximum Dips by Seismic Methods," pp. 171–172.
5. Sleight, E. R., "Development of Mathematics in Scotland, 1669–1746," pp. 173–185.
6. Dragoo, R. C., "Teaching the Calculus," pp. 186–193.
7. Dorwart, H. L., "Post-War Blueprint," pp. 194–196.
8. Court, A. N., "Perspective Triangles," pp. 197–198.

School Science and Mathematics
April 1945, Vol. 45, No. 4

1. Carnahan, Walter H., "Illustrating the Conic Sections," pp. 313–314.
2. Loomis, Hiram B., Pandiagonal Magic Squares on Square Bases and Their Transformations, pp. 315–322.
3. Mayor, J. R., "Book Clubs for Professional Reading," pp. 323–325.
4. Griffin, W. Raymond, "Theory of Iterated Trigonometric Functions," pp. 341–350.
5. Thatcher, Leonard, "Relations of Trigonometric Functions," pp. 365–366.
6. Nyberg, Joseph, "Notes from a Mathematics Classroom," pp. 372–375.

Miscellaneous

1. Bentley, E. M., "Number Experiences Units for Kindergarten Pupils," *Education*, 65: 464–465, April, 1945.
2. Brown, E. O., "Problem Unit: Buying a Car," *Education*, 65: 478–479, April, 1945.
3. Brownell, W. A., "When is Arithmetic Meaningful?" *Journal of Educational Research*, 38: 481–498, March, 1945.
4. Campbell, J. D., "Arithmetic in the Medieval World," *School (Elementary Edition)* 33: 531–535, February, 1945.
5. Campbell, J. D., "Rule Arithmetic," *School (Elementary Edition)*, 33: 611–613, March, 1945.
6. Campbell, J. D., "Units of Measurement," *School (Elementary Edition)*, 33: 707–712, April, 1945.
7. Cobb, F. E., "Appreciation Unit in Measures: the Story of Paper Bags," *Education*, 65: 485–487, April, 1945.
8. Conklin, C., "Arithmetic for which I Have no Use in a War Plant," *Education*, 65: 491–492, April, 1945.
9. Dexter, C. E., "Analysis of Written Problems in a Recent Arithmetic Series," *Education*, 65: 488–490, April, 1945.
10. Dorwart, H. L., "Character of Mathematics," *Scientific Monthly*, 60: 177–180, March, 1945.
11. File, C. M., "Improvement of Business Mathematics," *Journal of Business Education*, 20: 19–20, April, 1945.
12. Gallagher, J. E., "Insurance: An Informational Problem Unit," *Education*, 65: 470–474, April, 1945.
13. Heflin, H. B., "Arithmetic for Every Day," *Instructor*, 54: 14–15+, April, 1945.
14. Martin, C. W., "Peru Improves Its Arithmetic Instruction," *American School Board Journal*, 110: 31+, April, 1945.
15. McCann, B., and Dalrymple, C. O., "Teaching the New Arithmetic in a Third Grade," *Education*, 65: 493–496, April, 1945.
16. Miller, N., "Ancestry of Modern Mathematics," *Queens Quarterly*, 1: 22–30, February, 1945.
17. Miller, N., "More Exercises on Functions," *School (Secondary Edition)*, 33: 528–529, February, 1945.
18. Orr, A. G., "Drill on Vulgar Fractions," *School (Elementary Edition)*, 33: 527–530, February, 1945.
19. Platt, J. R., "Models as Aids in Calculation," *American Journal of Physics*, 13: 53, February, 1945.
20. "Putting Mathematics to Work," *New York Society for the Experimental Study of Education*, 1943 Yearbook, pp. 120–122.
21. Renfrow, O. W., "Background of Arithmetic," *Journal of the National Education Association*, 34: 66, March, 1945.
22. Rich, F. M., "Number Combinations Self-taught with Home-made Abacus," *Journal of Education*, 128: 88–89, March, 1945.
23. Rosenberg, R. R., "Inventory Test in Business Mathematics," *Business Education World*, 25: 447–449, April, 1945.
24. Sauer, M. E., "Number Work in the Ele-

mentary Grades," *School Executive*, 64: 50–51, March, 1945.

25. Shea, M. A., "Making Arithmetic Live Through the Medium of Informational Problems Units," *Education*, 65: 455–459, April, 1945.
26. Stevens, M. P., "Teaching Arithmetic: Some Important Trends in our Schools Today," *Grade Teacher*, 62: 54, April, 1945.
27. Stone, M. B., "Capitalizing Children's Experiences in Arithmetic," *Education*, 65: 497–502, April, 1945.
28. Ulrich, L. E., "Arithmetic Terminology," *Wisconsin Journal of Education*, 77: 326–327, March, 1945.
29. Upley, M. M., "Thrift: A Functional Problem Unit," *Education*, 65: 460–463, April, 1945.
30. Wilson, G. M., "What is Functional Arithmetic?" *Education*, 65: 466–469, April, 1945.

◆ EDITORIAL ◆

New Subscribers

TWENTY-FIVE HUNDRED copies of this issue of THE MATHEMATICS TEACHER are being mailed to prospective members of The National Council of Teachers of Mathematics whose names have been sent in by the various state representatives throughout the country. It is hoped that many of these people will become members of the Council. Now is the time for those of us who are interested in the improvement of mathematical education to put in some extra work in encouraging teachers to join the Council. The state representatives are doing a fine piece of work, but they cannot do everything. All of us should do our part in trying to get teachers of mathematics to become active members by subscribing to the official journal and to support the work of the Council by buying the yearbooks. At the end of this issue there appears a page of explanation of the work that the Council does and a subscription blank for the use of prospective members.

W. D. R.

NEWS NOTES

A General Meeting of the Mathematical Association was held at King's College, Strand, London, W.C.2, on Thursday, 5th April, 1945, and Friday, 6th April, 1945.

Thursday, 5th April, 1945

10:30 a.m. Business:—

(a) Joint Report of the Council and the Executive Committee for 1944.

(b) Election of Officers and the Council. Prof. S. Chapman, F.R.S., was proposed as President for 1945.
The existing Vice-Presidents, the Treasurer, Secretaries, Librarian, and the Editor of the *Mathematical Gazette* were proposed for re-election.
The existing Auditor and the present Members of the Council were proposed for re-election.

(c) The following new Rules were proposed by the Hon. Treasurer, Mr. K. S. Snell, and seconded by Mr. G. L. Parsons:—

Rule 32 (iv):

(A) The income and property of the Association, whencesoever derived, shall be applied solely towards the promotion of the objects of the Association as set forth in these Rules, and no portion thereof shall be paid or transferred directly or indirectly, by way of dividend, bonus or otherwise howsoever by by way of profit, to the members of the Association.

Provided that nothing herein shall prevent the payment, in good faith, of reasonable and proper remuneration to any officer or servant of the Association, or to any member of the Association in return for any service actually rendered to the Association.

Rule 33A:

If upon the winding up or dissolution of the Association there remains, after the satisfaction of all its debts and liabilities, any property whatsoever, the same shall not be paid to or distributed among the members of the Association, but shall be given or transferred to some other institution or institutions having objects similar to the objects of the Association, and which shall prohibit the distribution of its or their income and property among its or their members to an extent at least as great as is imposed on the Association under or by value of Rule 32 (iv) hereof, such institution or institutions to be determined by the Members of the Association at or before the time of dissolution, or in default thereof by a Judge of the High Court of Justice having jurisdiction in regard to charitable funds, and if and so far as effect cannot be given to such provision, then to some charitable object.

The purpose of the proposed new rules is to support the claim of the Association for exemption from Income Tax.

11:00 a.m. *Presidential Address* by Mr. C. O. Tuckey.
"Teachers and Examiners."

2:00 p.m.–4:15 p.m. *Discussion on Technical Mathematics.*
Opened by Dr. McLachlan.
"Undergraduate and Post-Graduate Technical Mathematics,"
and Mr. H. V. Lowry,
"Mathematics in Technical Colleges."

5:00 p.m. *Integrals in Infinitely Many Dimensions.*
Prof. P. J. Daniell, Sc.D.
In applying statistics to physical or mathematical sciences there is need to consider functions fluctuating erratically throughout their whole range of definition. These functions can be distinguished by tabulating either their Fourier coefficients or their values at all rational values of the variable. Development follows through Gaussian distributions and correlation coefficients. Applications have been made to the Kinetic theory of gases and to almost periodic functions. Statements about "nearly all" functions or "nearly all" gases acquire a meaning.

Friday, 6th April, 1945

10:00 a.m. *Some Mathematical Aspects of Punched Card Accounting Machinery and Methods.*

Mr. R. A. Fairthorne.
Machinery using data materialised as perforated or notched cards has been used for fifty years or so for statistical and accountancy work, and recently for scientific and technical computations. In the solution of the cardinal arithmetical problems it displays the ingenuity common to most computing machines, but its characteristic feature is the use made of ordinal ("administrative") relations in the data. Ordinal arithmetic is, at the moment, studied mainly as rule-of-thumb by filing clerks or as, say, Group Theory, Topology, or Combinatorial Analysis by pure mathematicians. Ordinal machinery makes very practical application of these abstract studies. For instance, the numerical coding of data can vary from arbitrary labelling to the search for isomorphic groups, and the capacity for dealing with combined ordinal and cardinal relations makes possible the use of "graphical" methods on data in purely numerical form.

11:00 A.M. *Mathematical Models and Constructions.*
Mr. A. P. Rollett.
The making of models of solids and surfaces linkages and miscellaneous apparatus. The construction of loci, envelopes and other drawings and graphs. The establishment of a "museum" of models and drawings; their use to the teacher and their value to the contributors.

2:00 P.M.–4:15 P.M. *Syllabuses for Examinations taken by Sixth Form Pupils.*
A discussion on the Syllabuses suggested by the Cambridge Advisory Committee and circulated to members: to be opened by Mr. K. S. Snell, followed by Dr. E. A. Maxwell and Mr. J. L. Brereton.

5:00 P.M. *Statistics and the School Course.*
Dr. J. W. Jenkins.
Elementary statistics will be presented as an integral part of a school course in mathematics related to social and scientific problems. Two stages are distinguished.
(A) An introductory stage in which measures of location and dispersion are derived from a study of frequency distributions, together with measures of association and correlation derived from tracing correspondences and general trends.
(B) A formal stage in which these basic ideas are systematised and methods developed for use in analysing fallible data.

The Programme Committee will be glad to receive at once any suggestions for next year's meeting. Such suggestions should be given, in writing, to one of the Secretaries.

Honorary Secretaries
G. L. Parsons,
Merchant Taylors' School, Sandy Lodge, Northwood, Middx.
(Mrs.) E. M. Williams,
Goldsmith's College,
at University College,
Nottingham.

The spring meeting of the Connecticut Valley Section of the Association of Teachers of Mathematics in New England was held on Saturday, April 14, 1945, at The Hotel Sheraton, Springfield, Massachusetts.

Program

Morning Session

10:30 "We Like To Teach It This Way"—Mrs. Helen M. Roberts, University of Connecticut, Storrs, Connecticut.
"Visual Aids in Mathematics"—Mr. Holmes Boynton, New Haven State Teachers College, New Haven, Connecticut, and Mr. J. Whitney Colliton, Rutgers University, New Brunswick, New Jersey (Formerly Central High School, Trenton, N. J.).
1:00 Luncheon

Afternoon Session

2:30 Business Meeting
2:45 "Mathematics Films"—Mr. Boynton and Mr. Colliton, assisted by the Audio-Visual Aids Center of the University of Connecticut.

Officers of the Connecticut Valley Section:
Levings H. Somers, *President,*
Pomfret School.
Elizabeth A. Harkness, *Vice-President,*
Northampton High School.
George E. Frost, *Secretary,*
Holyoke High School
Ethelyn M. Percival, *Treasurer,*
Westfield High School.
Laura D. Sargent, *Director,*
Ethel Walker School.
Neal H. McCoy, *Director,*
Smith College.

The sixth meeting of the Men's Mathematics Club of Chicago was held on April 20, 1945 at the Central Y.M.C.A. at 19 S. La Salle St. The seventh meeting of the club was held at the same place on May 18, 1945. The two programs follow:

At the Sixth Meeting Dr. E. P. Northrop of the University of Chicago spoke on the topic "Mathematics in Liberal Education" and Mr. J. A. Nyberg spoke on the topic "A Pre-induction Course in Mathematics for Seniors."

At the Seventh Meeting Professor H. T. Davis of Northwestern University spoke on the topic "Computing is a Fine Art."

The following officers were elected for the 1945–1946 season:
Prof. H. T. Davis, Northwestern University, Evanston, Illinois. Honorary President.
Walter W. Barczewski, President, Waukegan Township High School, Waukegan, Illinois.
Glenn Anderberg, Secretary-Treasurer, Waukegan Township High School, Waukegan, Illinois.
H. C. Torreyson, Recording Secretary, Lane Technical High School, Chicago, Illinois.

The Committee on Teacher Education, the American Council on Education's newly appointed group charged with the responsibility for further implementing the work and findings of the Commission on Teacher Education, announced this week the appointment of Dr. L. D. Haskew as its Executive Secretary. Offices for the committee will be located at 525 W. 120th St., New York 27, N. Y., where Dr. Haskew, who is on leave from his position as Director of Teacher Education at Emory University, will assume direction of the Committee's program on April 1.

The Committee on Teacher Education plans to devote its major attention to assisting school systems, institutions, and organized agencies with problems involving the recruitment and education of teachers, bringing to bear upon those problems the experience of the Commision on Teacher Education and its professional staff. Several volumes reporting and analyzing the Commission's experiences are already available, and additional publications are scheduled to appear this year. Those already published are: ***Teachers for Our Times; Evaluation in Teacher Education; Teacher Education in Service;*** and ***The College and Teacher Education.***

Membership for the new Committee on Teacher Education has been drawn chiefly from the former Commission on Teacher Education. Chairman is Professor E. S. Evenden, Teachers College, Columbia University, and other members are: Professor Karl W. Bigelow, also of Teachers College; Professor Russell M. Cooper,

University of Minnesota; Professor Mildred English, Georgia State College for Women; President Charles W. Hunt, Oneonta (N. Y.) State Teachers College; Dr. A. J. Stoddard, Superintendent of Schools, Philadelphia; Dean Ralph W. Tyler, University of Chicago; and President George F. Zook, American Council on Education.

WOMEN'S MATHEMATICS CLUB OF CHICAGO AND VICINITY

President, MISS MARION ECKEL, Kelly High School.
Vice-President, MISS MARIE GRAFF, South Shore High School.
Secretary, MISS VIRGINIA TERHUNE, Proviso Twp. High School.
Treasurer, MISS EDITH LEVIN, Englewood High School.
Program Chairman, MRS. ELSIE P. JOHNSON, Oak Park High School.

The Annual Joint Meeting of the Men's and Women's Mathematics Clubs was held at 6:30 P.M., Friday, March 16, 1945, at Huyler's, 308 South Michigan Avenue.

The annual joint meeting of these two clubs is the high spot of the year's program.

Dr. Mayme I. Logsdon, Associate Professor of Mathematics at the University of Chicago and author of "A Mathematician Explains," answered the question: "What Shall We Do Now?"

Dr. M. L. Hartung recently wrote us saying "Just finished reading the March MATHEMATICS TEACHER from cover to cover. Every article has something, and several have a lot, to offer, Wheat and Snader especially good. Nice going!"—Editor

The Metropolitan New York Section of The Mathematical Association of America held its Fourth Annual Meeting at The Polytechnic Institute of Brooklyn, 99 Livingston Street, Brooklyn, N. Y., on Saturday, April 21, 1945, at 10 A.M.

Program

Chairman: Professor Jewell Hughes Bushey, Hunter College

"Demonstrative Algebra"—Professor E. R. Stabler, Hofstra College
"Triangular Permutations"—Mr. John Riordan, Bell Telephone Laboratories
"Hadamard's Determinant Theorem"—Professor John Williamson, Queens College
"Changing Objectives in the Teaching of Mathematics in the Senior High School":
A. "Algebra and Trigonometry"—
Mr. Benjamin Braverman, Seward Park High School
B. "Geometry"—
Mr. Samuel Welkowitz, Franklin K. Lane High School

Officers of the Metropolitan New York Section, 1944–1945:

JEWELL HUGHES BUSHEY, Hunter College, Chairman
NATHAN LAZAR, Midwood High School, Vice-Chairman
HOWARD E. WAHLERT, New York University, Secretary
FREDERIC H. MILLER, Cooper Union, Treasurer

The following letter should be of interest to our readers:

Rushville, Indiana
May 12, 1945

THE MATHEMATICS TEACHER
525 W. 120th St.
New York 27, N. Y.

Dear Sirs,

Please find enclosed check for $2.00 for renewal of my subscription. I have certainly enjoyed and appreciated the articles in the past year.

I was pulled partially away from my insurance business three years ago to do emergency mathematics and physics teaching. I had been away from the classroom for fifteen years. Your magazine has helped me a great deal to get up to date. I will probably wish to continue my subscription after the emergency is over and I once more retire from active teaching.

Sincerely,
Dudley Campbell